SELECTION OF MULTICRITERIA DECISION MAKING METHODOLOGIES IN SCENARIO BASED PLANNING

by

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SYNOPSIS

This dissertation investigates the application of Multicriteria Decision Making (MCDM) methodologies to the area of scenario based policy planning. We examine how the tools of MCDM can be used to develop a Decision Support System (DSS) that would allow management or policy planners to resolve conflicting goals and interests. Ideally, the resolution would be obtained by the various decision makers (DMs) in such a manner, that it satisfies all the relevant interest groupings at a maximum level of achievement for all concerned. This is not always possible and compromises need to made that are fare and equitable to all the relevant interests.

Stewart *et al.* (1993), in a report entitled: "Scenario Based Multicriteria Policy Planning for Water Management in South Africa", develop the principles of a procedure for implementing scenario based multicriteria policy planning. Their iterative procedure is illustrated in figure 2.1, chapter 2, of this paper. In this dissertation, we refine certain parts of this procedure and the two areas in particular that we have looked at are:

(1) filtering a large set of policy scenarios (Background Set), that could be a continuum, to form a smaller set (Foreground Set), and

(2) further reducing the smaller set to form a solution set of policy scenarios.

(The generic terms "Background Set" and "Foreground Set" are defined in section 2.1 of chapter 2.)

The main objectives of this study were therefore mainly twofold and are as follows:

(1) to determine what MCDM methods are relevant to natural resources management (using water as a case study), and

(2) to investigate how these methods need to be adopted for use in an interactive DSS.

We address the first objective by surveying the literature in an attempt to identify potential MCDM approaches that are suitable to (i) reduce a large set of alternatives, analogous to the Background Set, to a more manageable smaller set, analogous to the Foreground Set of alternatives, and (ii) refine this Foreground Set in order to present the DMs with a solution set of alternatives from which they will make their final selection. The literature has until now not dealt explicitly with these two issues and we had to adapt certain MCDM approaches, many of which have been developed in a linear programming context, to suit our purposes.

Chapter 1 of this dissertation therefore sketches the background to this study, by providing the motivation for this work and setting out the manner in which the rest of this dissertation will follow. Chapter 2 deals with the literature review that was undertaken. Here we provide the detailed procedure developed by Stewart *et al.* (1993). The chapter also explains some of the nomenclature that has been or will be used in the remainder of this dissertation. The two remaining (main) sections of this chapter concentrate on reviewing the appropriate MCDM approaches that could be applied to firstly, the Background Set, and secondly, the Foreground Set of alternatives.

Chapter 3 provides the reader with a description of the approach that was adopted for the simulation studies in this dissertation. These studies were used to analyze selected MCDM approaches in an attempt to evaluate the overall quality of the solutions that they produce. The chapter provides the reader with summaries of both data sets that were used in the simulation studies and includes the experimental design technique that was used to generate the second data set. Chapter 3 further outlines the various stages of the simulation algorithm, with the algorithm itself being illustrated, for a particular iteration of a simulation run, diagrammatically in figure 3.1. The simulation algorithm was applied separately to each of the two data sets and the results obtained from these simulation runs were reported in chapters 4 and 5.

Chapter 4 reports the results that were obtained from the first data set, and includes the sensitivity runs that were conducted on the various MCDM approaches. Chapter 5 reports the results that were obtained from the analysis conducted on the second data set, including the sensitivity analysis conducted on the various MCDM approaches, and the effects that external factors such as the number of policy elements, derived attributes or interest groups, have on these results.

The conclusions that were reached, based on the findings of the analysis as reported in chapters 4 and 5, regarding

(1) which methods provided the most consistent and reliable solutions, as well as

(2) the general value of MCDM methods to multicriteria policy planning in water resources management

are then finally reported on in chapter 6. This chapter therefore serves as a concluding chapter to this dissertation.

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CHAPTER 1: INTRODUCTION

1.1 Background to the Problem

Where one finds humans, one is bound to come across problems. This is not necessarily a bad reflection on the human race as such, but serves as a reminder of the realities that have, and will continue to, characterize the history of humanity. These realities are embodied in the numerous problems people face today, some of which are latent, others have been present since time immemorial. However, the human race has not only become synonymous with problems. Accompanying these problems, humans have, or at least are attempting to, find solutions. It can of course, quite justifiably, be argued that certain (complete) solutions still remain latent. An opposing and perhaps more optimistic view can, however, be taken. It would state the following: Where one encounters the human race, one is bound to come across solutions. These solutions are merely waiting to find the problems to which they can be applied.

One of the problem areas people face today, is that of managing their scarce natural resources. But why bother? The answer to this is quite simply, that in order to survive as a species, humans need to preserve and effectively manage their environment. The manner in which this can be achieved, remains a much debated argument. However, humanity is faced with depleting natural resources on the one hand, and competing users of these scarce resources on the other. Humans have to make informed decisions on how adequately to manage these resources and thereby satisfy all the competing users thereof.

The problem or scenario described in the paragraph above, is one that involves a decision maker (DM), or group of decision makers, contemplating a choice of action. This choice has to be made from a number of alternatives, where each alternative produces one or more, possibly uncertain, outcomes. For certain problems, the alternatives are relatively easily defined whilst for others they still have to be generated. It is, however, apparent that this problem will have multiple objectives associated with it, these objectives stemming from the competing users of the natural resource(s). Often these objectives are in conflict with one another. This conflict has to be resolved in a way that satisfies the requirements of all the competing users, both fairly and equitably. The science of decision analysis deals with the resolution of this type of conflict, although it must be said that it does not always provide an explicit solution to the problem being investigated. As Keeney (1982) puts it:

> "Decision analysis will not solve a decision problem, nor is it intended to. Its purpose is to produce insight and promote creativity to help decision makers make better decisions."

1.2 Statement of the Problem

We have recognized in section 1.1 that where decisions need to be taken that involve multiple, non–commensurate, conflicting objectives, a fair and equitable solution satisfying all parties concerned becomes a primary aim. Furthermore, planning and managing the use of natural resources becomes a complex process, that requires taking decisions inevitably involving several objectives of an economic, environmental and social nature.

Multiple Criteria Decision Making (MCDM) techniques can play an important role in analysing such problems involving several objectives. Human value judgements, conflicting interests and considerations other than economic efficiency can and should be integrated within a single model. This permits the DM(s) to better understand the possible trade–offs among the various objectives, and then to apply value judgements in arriving at a conclusion.

The issue that now needs to be addressed is, to determine which MCDM methods are relevant to particular situations, i.e. situations involving the management of natural resources. In addition to their relevancy, the issue of how adaptable and implementable these MCDM methods are to various situations, also needs to be addressed.

1.3 Motivation of the Problem

Most water resources planning and management problems are so complex as to preclude the possibility that any individual, or group of DMs and analysts, can assess the implications of the decisions to be made, especially with multiple conflicting objectives. This is why models and multiobjective decision making methodologies are needed. The same difficulties are encountered, and solutions applied, to all other natural resources planning and management problems.

However, the process of analysing the problem also does not guarantee an explicit solution(s) – see Keeney's comment in section 1.1. What is true, is that the analysis enables the DM(s) to gain an increased understanding of the decision problem. The insights gained may suggest other approaches to solving the problem or lead to a greater common understanding amongst a heterogeneous group of DMs.

As stated earlier in this chapter, considerations other than economic efficiency need to be addressed when dealing with resource allocation problems. Kindler (1982) states the following:

> "...there is a common tendency to pursue economic optimality alone in water-resources allocation, as though there were some inherent nobility in this goal."

He continues to say that not only is the pursuit of the goal of economic optimality often not attainable, but that it is also quite often inadvisable. Kindler, however, never completely discards the pursuit of economic efficiency, as is clear from the trade-off he makes between economic factors and other factors. He further states:

> "...It does not mean, however, that there is something inherently wrong with the economic efficiency objective. ...The solution of allocation problems in real life, must focus on institutional and other objectives in addition to that of economic efficiency."

Multiobjective or multicriteria decision making methodologies can and do address these concerns.

1.4 Objectives of the Study

In a report entitled: "Scenario Based Multicriteria Policy Planning for Water Management in South Africa" (Stewart *et al.*: 1993), the principles of a procedure for implementing scenario based multicriteria policy planning has been developed. However, certain areas of this procedure need further refining and it is these areas upon which this paper will focus. The two areas in particular that were looked at are:

(1) filtering a large set of policy scenarios (analogous to the Background Set we define in chapter 2), that could be a continuum, to form a smaller set (analogous to the Foreground Set we define in chapter 2), and

(2) further reducing the smaller set to form a solution set of policy scenarios.

The main objectives of this study are therefore:

(1) to determine what MCDM methods are relevant to natural resources (particularly water) management (in South Africa), and

(2) to investigate how these methods need to be adopted for use in interactive decision support systems (DSS).

Concerning the first objective above, the study will focus on how a large set (that could essentially be a continuum) of policy options (equivalent to the "*Background Set*" of policy options as defined by Stewart *et al.* (1993) - for the definition of this generic term, see section 2.1 of chapter 2) is reduced to a more manageable smaller set. The reasons for doing so would be that it is more practical, and from a cognitive perspective, less stressful and therefore easier to perform direct value judgements on such a reduced number of policy options. This is further discussed in the literature survey of this report. Furthermore, once the smaller set has been obtained, direct value judgements still have to be carried out on the policy options in order to select a solution set of alternatives. The MCDM methods facilitating this process will also be investigated.

This paper will also report on some of the limitations of the methods that were encountered as well as how they need to be adapted for use in an interactive DSS.

1.5 Limitations of the Study

Due to the large costs involved in reconstructing a decision event such as, say, the management of a river system or catchment area, the approach adopted in this study is that of utilising a simulation study. There are decided advantages to using such an approach, for example, the minimal costs this approach would incur. However, there are also decided disadvantages that could stem from this type of approach. Some are referred to below.

First, for practical reasons, it was decided that the manner in which the conflict between the divergent views, as represented by the different DMs, was resolved, would only be simulated as follows: We would allow for one attempt at reducing the large set of policy alternatives to a smaller set and then allow for only one further attempt to find a solution set consisting of one or more alternatives from this smaller set. The first attempt therefore involved the extraction of a smaller set of policy alternatives, termed the "*Foreground Set*" by Stewart *et al.* (1993) (this generic term is defined in section 2.1 of chapter 2). The second attempt allowed for a revision of this smaller set before choosing a solution set consisting of one or more alternatives. Practical constraints had to be considered when making this decision and these will be discussed later in this paper. However, by limiting the re-evaluations conducted on both the large and the small sets, we in a sense fail to capture the true essence of what the consensus seeking process, an essentially iterative process, fully entails, when solutions to divergent objectives are being sought.

A second limitation faced with the simulation approach was that of time. This was not as great a limitation as it would have been, had an actual real life case study been reconstructed. Time was only an issue because we did not limit our study to only one MCDM or related method. We included many, and also investigated combinations of methods. This of course enabled us to form a much better picture of how effective the methods were. We will deal with this in more detail once the results of the analysis have been reported on.

1.6 Plan of Development of Report

Chapter 2 (a literature review) of this paper begins by providing one with a brief background to the simulation study and explains how it evolved from the report by Stewart *et al.* (1993). In this chapter we discuss which MCDM and related methods are used for finding solutions to the two main problem areas focused on in this paper. These problem areas as stated before are (i) that of reducing a large set of policy options to a more manageable smaller set, and (ii) that of refining the smaller set of options (using full or partial rank orderings) to find the best policy(s).

Chapter 3 provides a full description of the simulation approach adopted for this study. This will include the actual methods used, the manner in which final policy options were evaluated as well as the technique of using "thermometer" type scales to resolve conflict between various interests with divergent objectives. This chapter will also discuss the two data sets used and provides full details of the first of these data sets. The second set varied from one iteration in the simulation process to the next and therefore the experimental design procedure, used to generate this varying set, is dealt with at length.

Chapters 4 and 5 describe the overall results of the analysis conducted on the two data sets. Both chapters include the results that the different methods or combination of methods yield, as well the results from the sensitivity runs conducted on the different methods or combination of methods.

Chapter 5 also examines the effects that external factors, for example the number of policy elements, have on the results produced by the different methods.

The conclusions that were reached, based on the findings of the analysis, regarding

(1) which methods provided the most consistent and reliable solutions, as well as

(2) the general value of MCDM methods to multicriteria policy planning in water resources management

are then finally reported on in chapter 6, thereby serving as a concluding chapter to this dissertation.

CHAPTER 2: LITERATURE SURVEY

2.1 Background to the Study

MCDM, as a formal approach to problem solving, involves human judgement, and can never be automated as a process by means of strictly adhered to algorithms or problem solving techniques. Human judgement, is of course what all decisions depend upon. This includes judgement about uncertainty, judgement about which alternative courses of action are available, judgement about the possible outcome(s) of a course of action and judgement about preferences. MCDM methodologies will not replace or automate these judgements; instead it will "provide a framework which will help decision makers to clarify and articulate them", as stated by Goodwin and Wright (1991). This framework will, according to French (1989), provide for the modification and, indeed, evolution of beliefs and preferences of DMs. French also states that the central role of decision–analytic models, in structuring discussion and helping group communication, remains largely unrecognized.

Having recognized the role that MCDM methodologies can play in facilitating decision making processes, we still need to develop a framework for incorporating these methods into planning procedures. We also have to evaluate the appropriateness of a variety of methods to certain decision situations. In the report by Stewart *et al.* (1993), entitled: "Scenario Based Multicriteria Policy Planning for Water Management in South Africa", the authors define a procedure for implementing scenario based, multicriteria policy planning in order to accommodate the needs of water management in South Africa. They do, however, point out that this procedure is not limited only to water management and state:

> "...the results would be generally applicable to a wide variety of other resource management problems, wherever there exists the need to take cognizance of conflicting and sometimes intangible interests of society."

The full scenario based planning procedure, as developed by Stewart *et al.* (1993), is illustrated diagrammatically in figure 2.1 on the following page (in their report it was figure 4.1). A key part of the procedure is that it is an iterative process involving a continual refinement of the solution, before the final recommendations can be made.

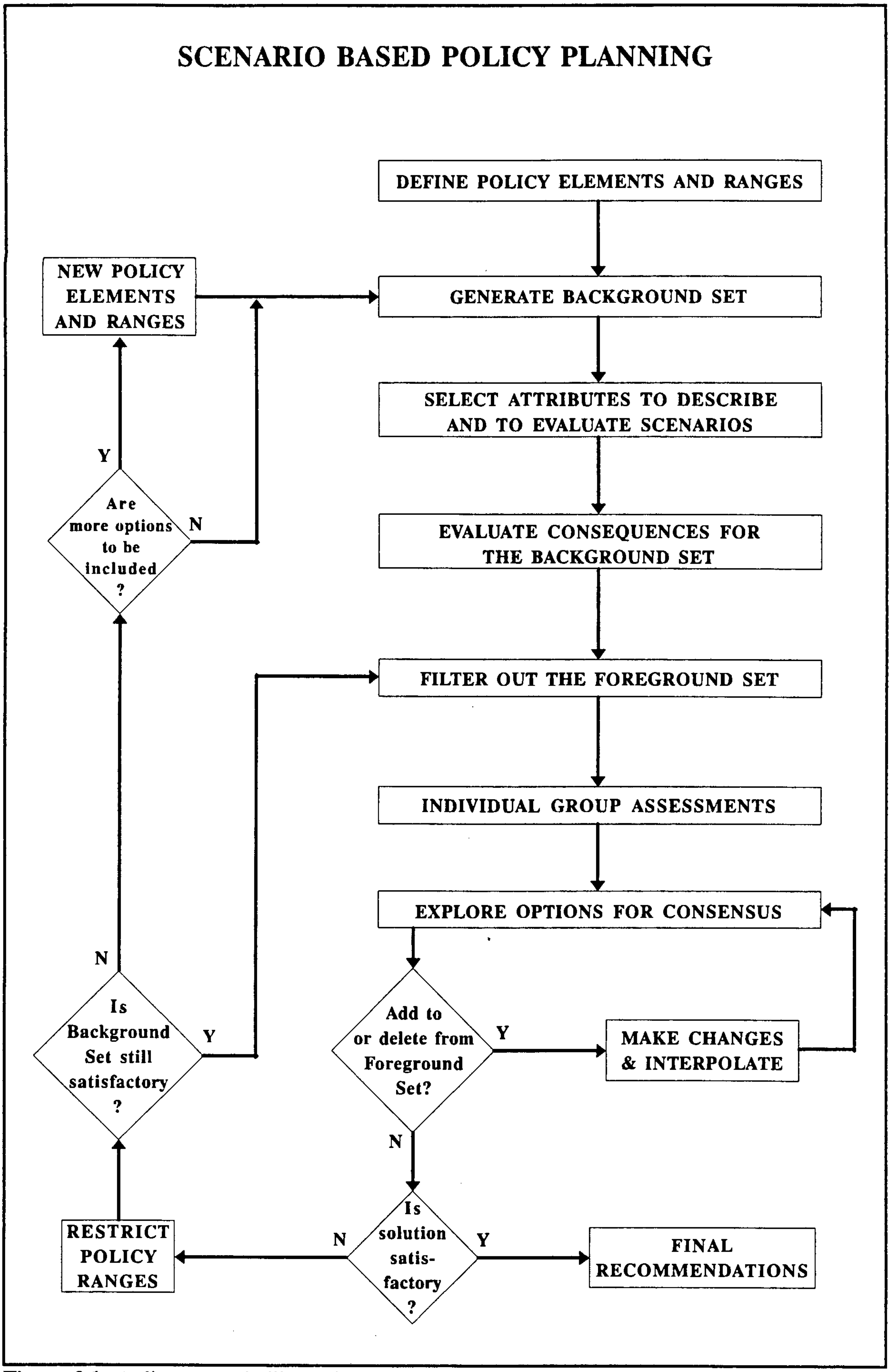

Figure 2.1: A diagrammatic illustration of the scenario based policy planning procedure

Shamir (1983) states that practical methods for multiobjective decision making must be both "interactive and iterative". He adds that they should provide results at each stage of the iterative analysis. These results must be provided in a form that will enable the DMs to "formulate their responses to these outcomes, and to incorporate their instructions into the next phase of the analysis".

In the remaining sections of this chapter (and for the remainder of the report) we will use the generic terms "Background Set" and "Foreground Set" in the manner that they have been defined by Stewart *et al.* (1993). These definitions are basically as follows:

Background Set: A pool (or set) of scenarios that is sufficiently rich so that all interested parties can find a reasonably satisfactory alternative, perhaps by interpolation between alternatives. This may simply be the full decision space, but in cases where implications of alternatives require extensive modelling or analysis, it may be necessary to work only with a representative but discrete set of options. The latter is typical of resource planning problems, for example water.

Foreground Set: A selection of scenarios (typically 5 to 9) from the Background Set on which direct value judgements are made by interested parties. These value judgements are expressed in terms of comparisons between the scenarios in the Foreground Set.

Furthermore, to clarify some of the more frequently used terms in this report and for the purposes of exposition, we present some definitions below.

Attribute (System Attribute): A measurable property of a system that can serve as a basis for comparing outcomes of different policy actions, and which can in principle be predicted for any proposed policy. It is a measure that evaluates the achievement of goals and objectives.

Criterion: A basis (or tool) for comparing or evaluating different decision or policy options according to a particular significance axis or point of view.

Objective (Goal): An operational definition of a particular criterion, expressed either as a direction of increasing preference (objective), or as a desired level of achievement (goal), in terms of the appropriate system attribute or performance measure.

***Surrogate Planning Objective*:** Objectives representing the types of aspirations that can be expected to be found amongst interest groups. They could consist of selected policy elements, or attributes, or simple functions of these.

***Policy Element*:** Individual instrument or component of a resource management policy (eg. water) that might be considered.

It is generally assumed that each criterion can be represented by a surrogate measure of performance, represented by some measurable attribute of the consequences arising from the implementation of any particular decision alternative (or policy scenario). We therefore specify that for each alternative a, we can associate a vector of attributes $\mathbf{z}^a = (z_1^a, z_2^a, ..., z_k^a)$ for k criteria. z_i^a will then be the attribute representing the outcome of decision alternative a as it affects criterion i. We will also assume that the DM prefers larger to smaller values of each attribute z_i, all other things being equal.

Before continuing, we assume for the purposes of our discussion, that all policy elements and ranges, as well as attributes used to evaluate the policy scenarios (decision alternatives), already exist. The terms "decision alternatives" and "policy scenarios" will be used interchangeably in the report. The process of identification of policy elements and ranges is best achieved by means of brainstorming techniques involving relevant policy makers and interest groups. As an example the reader is referred to Delbecq *et al.* (1975), wherein the Nominal Group Technique (NGT) which is well suited for this purpose, is defined.

The identification of system attributes, that measure goal (objective) achievement by means of evaluating the policy scenarios, is also well documented in the literature. The common assumption in MCDM is that a hierarchy of criteria can be constructed, so that at the lowest level, the objectives (or goals) are expressed as maximising or minimising some well defined system attribute. There are desirable properties that such a set of criteria should satisfy. According to Keeney and Raiffa (1976), the criteria should be:

Complete:	i.e. they should cover all aspects of the problem.
Operational:	i.e. they can be meaningfully used in the analysis.
Decomposable:	i.e. they can be broken into parts to simplify the process.
Nonredundant:	i.e. they avoid problems of double counting.
Minimal:	i.e. the number of attributes should be kept as small as possible.

The reader is, however, referred to Keeney and Raiffa (1976), for a more detailed discussion on these desirable properties.

The generation of policy scenarios or decision alternatives is a non-trivial task. It is not even clear as to whether the set of policy scenarios can be pre-defined before the start of an analysis. However, this set could in effect be a continuum of policy alternatives. This may make it an impractical set to work with since its size is so large, and this is why we might work with a representative discrete set of alternatives, such as the Background Set (as defined earlier in this section). In this study, an adapted form of an experimental design procedure was used to generate a Background Set from an effectively continuous decision space. The experimental design procedure will be further reported on in chapter 3.

Our discussion for the remaining two sections of this chapter will focus on reviewing MCDM and related approaches that are relevant to large decision spaces (section 2.2), analogous to the Background Set of alternatives, as well as those approaches that could be used to analyze a reduced set of alternatives (section 2.3), analogous to the Foreground Set.

2.2 A Review of MCDM Approaches Relevant to Large Decision Spaces

In this section we will be reviewing some MCDM and related approaches that can be applied to a set of policy scenarios under consideration. This set will typically be a large, but finite set of alternatives forming our decision space, analogous to the Background Set we have defined in the previous section. We discuss the relevant MCDM approaches under their broad headings below.

(a) Utility or Value Functions

In multiattribute utility theory (MAUT), also termed multiattribute value theory (MAVT), a global utility or value function (as developed in, say, Keeney and Raiffa (1976)) is established that represents the overall strengths of preference of the DM between outcomes. According to Duckstein *et al.* (1982), utility is defined, within the context of planning, as the subjective benefit(s) derived by the DM from the achievement of the stated goals or objectives. The utility function can be specified numerically by eliciting the DM's utility for each criterion (marginal utilities $u_1, u_2, ..., u_k$ for k criteria), and then combining these into a global utility function $U(u_1, u_2, ..., u_k)$. Typical forms of the utility function are:

(1) the additive form:

$$U(u_1, u_2, ..., u_k) = \sum_{i=1}^{k} w_i u_i$$

and

(2) the multiplicative form:

$$1 + KU(u_1, u_2, ..., u_k) = \prod_{i=1}^{k} [1 + Kw_i u_i]$$

for some parameter K.

For the additive form to be able to model the DM's strengths of preference consistently, the assumption of *additive difference independence* must hold. This assumption states that absolute strengths of preference between two outcomes differing only on one criterion, do not depend on the levels of achievement on the other criteria. For the multiplicative form to hold, the assumption of *preferential or weak difference independence* must hold. It requires that relative strengths of preference between scenarios differing on one criterion only, are independent of levels of achievement on the other criteria.

The marginal utilities (u_i) can be evaluated indirectly by the standard decision analysis technique of ascertaining certainty equivalents for two–point lotteries, or by direct scoring. The reader is referred to Keeney and Raiffa (1976), for a detailed discussion. The weights (w_i) can be determined directly by various weighting techniques. Some techniques, together with the references that describe them are: the ratio method in Edwards (1977), the swing–weights method in von Winterfeldt and Edwards (1986), and the tradeoff and pricing out methods both in Keeney and Raiffa (1976).

The process of assessing both the marginal and the global utility functions can be tedious and time consuming when these functions are assessed, as is done in classical MAUT, by means of (DMs) evaluating sequences of hypothetical alternatives to find points of indifference. The policy scenario that provides the highest global utility value is defined to be the preferred solution. The remaining scenarios are ranked according to their global utility values, and these rankings would indicate which scenarios could form part of a short-list of "good" alternatives. The process of going through the exercise of assessing the value functions and the weights creates considerable insight into the problem. This insight can be far more important than the mechanical ordering of the policy scenarios, and may also play an important role in the process of further reducing the list of alternatives.

(b) Value Functions in Interactive Mode

Value functions require of the DM to specify preferences over wide ranges (of criteria). These preferences must be specified *a priori*. By placing a restriction on the ranges for which preferences must be specified, we make it easier for the DM. There are various techniques that adopt this approach. Within the context of multiple objective linear programming (MOLP), we will discuss some of these interactive techniques, based on the "addition of scores" approach. These scores represent some goal (objective) achievement in terms of a level of performance for a particular criterion.

The first set of techniques can be divided into three broad algorithms. In the Zionts' algorithm (1976), the problem is discrete and the underlying utility function is linear. In the Zionts and Wallenius approach (1976, 1983), the problem is continuous and the underlying utility function is linear. This method is extended to allow for a concave utility function. Finally, in the Korhonen, Wallenius and Zionts approach (1984), the problem is discrete and the underlying utility function is assumed to be quasi-concave. All three methods generate a composite linear function of the form shown below:

$$\sum_{i=1}^{k} \lambda_i z_i$$

The weights λ_i, are generated arbitrarily to start with, and the solution (or alternative) that maximizes this function is found. The DM is then asked to choose between the maximizing (or reference) solution and a sequence of solutions, one at a time. Based on the DM's responses, a consistent set of weights is chosen and a new maximizing alternative is found in the Zionts' algorithm (1976). If the DM prefers this new alternative to the reference alternative, it becomes the reference alternative. Otherwise, an alternative preferred to the old reference alternative becomes the reference alternative. The process continues until no alternative is preferred to the current reference alternative. At that time, some limits on the optimal solution are constructed, and any new alternatives not satisfying these limits are discarded from further consideration. If the number of remaining alternatives is sufficiently small, the procedure terminates and the remaining scenarios will form our Foreground Set. If not, the method is repeated. The Zionts' algorithm (1976), attempts to determine a "most preferred" subset of scenarios from a larger set, but not to generate a full preference ordering of alternatives, according to Khairullah and Zionts (1980).

In the approach of Zionts and Wallenius (1976, 1983), the DM is offered some trades from the original reference solution. These trades take the form: "Are you willing to reduce objective 1 by so much in return for an increase in objective 2 of a certain amount, an increase in objective 3 of a certain amount, and so on?" The DM is asked to respond either yes, no, or I don't know to the proposed trade. The method then develops a new set of weights consistent with the responses obtained, as well as a corresponding new solution. This process is repeated until a best solution is found. The method may be extended to allow for the maximization of a concave function of objectives by making two changes, *viz.*: (i) the trades are presented in terms of scenarios: for example, "Which do you prefer, alternative A or alternative B?"; (ii) each new non-dominated extreme point solution of the problem is compared with the old, and either the new solution or one preferred to the old one is used for the next iteration. Finally, the procedure terminates with a neighbourhood that contains the optimal solution.

In the approach of Korhonen, Wallenius and Zionts (1984), the DM's answers, when making the choice between the maximizing solution and the adjacent efficient solution (Zionts (1981)), are used to generate a convex cone (or a set of cones) that is then used to eliminate solutions dominated by it. The individual's answers are also used to construct a new set of consistent multipliers (weights) and to find a new solution. The process is then repeated by identifying the adjacent efficient solutions to the new solution, asking new questions of the DM, and so on. The process will converge to an overall optimal solution with respect to the individual's implicit utility function.

A further technique is that of Steuer (1986), termed the interactive weighted-sums method. His method generates only a relatively small number of non-dominated extreme point solutions (scenarios). This is achieved by selecting a convex cone in λ (weights) space that in general is large and includes convex cones corresponding to many non-dominated extreme point solutions. Rather than generating all of them, he generates only a very small number of extreme point solutions, and questions the DM regarding their relative attractiveness. These responses he will use to contract the cone. A filter device is used to keep the number of solutions considered by the DM at a time as small as the DM wants.

These interactive value function based methods were developed in a linear programming context, but can in principle be applied whenever the space of feasible attribute vectors is convex. They tend to lend themselves well to the issue of reducing a large set of alternatives to a smaller set. Their interactive nature, and more so the reduced amount of comparisons needed, say, compared to the MAUT approach, are factors that make them particularly attractive for this purpose.

(c) Goal Programming and Related Techniques

The goal programming approach (attributed to Charnes and Cooper (1961)), in general, starts by having the DM specify achievement levels for each criterion, in terms of the relevant performance measures. These levels are, according to Stewart *et al.* (1993), typically one of three types. They are either (i) goals or aspiration levels, or (ii) veto or exclusion levels, or (iii) reference levels.

We suppose that for each attribute i (representing a particular criterion of evaluation), the DM can specify some desirable goal or target level of achievement, z_i^*, say. The aim of goal programming is to find a solution (alternative) which is as near as possible to the target. Some measure of the discrepancy that exists from the target needs to be defined. There are three possible forms of representing the discrepancy that have been proposed, and they can be used either singly, or in combination with each other. The three forms are (i) Archimedean, (ii) Pre-emptive, and (iii) Tchebycheff or Min-Max. For a discussion about these three forms, the reader is referred to Stewart (1992). The measures of discrepancy for each attribute are summed, and goal programming would then be used to minimize this summation function.

Goal programming has an advantage over utility based procedures when the number of criteria become excessively large. According to Stewart (1992), "goal programming is probably the method of choice for the purpose of screening a large (or infinite) number of alternatives down to a short–list, when the number of criteria is large". Furthermore, in real life situations, goal setting is a common phenomenon. Therefore, a technique that attempts to make the DM come as close as possible to his or her pre–specified goals, would seem an appropriate method to use for most DMs.

One of the major weaknesses of goal programming is that the DM must specify both goals and their relative importance (in terms of weights) *a priori*. The subjectivity inherent in determining the level of attainment for each goal, and the penalty weights for overattainment or underattainment, present a critical problem in the formulation of the goal programming model. It is therefore apparent in the case where goal programming techniques are applied to problem solving, that they should be used in an interactive mode.

The Interactive Multiple Goal Programming (IMGP) procedure of Spronk (1981) is a technique that was especially designed for financial planning, but which has been, and continues to be, used in a variety of different applications. The method works in terms of two sets of reference levels, where these levels converge towards each other. The "*nadir*" values are the set of lower bounds and the "*ideals*" represent the set of best achievable values or "potentials". At each iteration, the DM examines the lower bounds and indicates which of these should be improved first. Once an increase in the lower bound for this criterion has been made, the effect on the potentials, with regard to the now tightened constraint on acceptable values, is calculated and shown to the DM. If the DM agrees that the loss in potential is acceptable, the new lower bound is made definitive and the next phase of the interaction starts. If the DM does not accept the loss in potential, the increase (in nadir value) is reduced systematically until the losses in potential are acceptable.

The DM can continue with this procedure until he or she ends up with an environment containing the optimal solution. However, the DM can also stop the procedure sooner. This will occur as soon as there is a set of solutions satisfying the required goal values. The DM may then choose freely from this set and according to Spronk (1990), may even use viewpoints (and criteria) that were not (or could not be) included in the original model when making his or her choices.

The ideas of goal programming, and particularly those of goal programming techniques used in an interactive mode, makes this a very useful and transparent procedure to use when reducing a large number of alternatives to a smaller set. Interactive procedures induce learning effects and thereby allow DMs to correct their earlier sources of error, before they continue their search for a solution(s). Stewart (1992), reports to have found the IMGP procedure to be relatively slow in converging to an environment containing the optimal solution. However, he has found the procedure to produce very satisfying results and notes that a particular useful property of IMGP, is that the DM is not required to sacrifice anything which he or she may perceive they have already gained. Stewart (1988), reports that although progress towards a final solution was slow using the IMGP procedure, the result in the form of a final short-list (of alternatives) was preferred to a single solution. He also states that the user expressed satisfaction with IMGP, in that it was "easy to understand", and that "continual progress" was seen to occur (towards the final solution).

Stewart *et al.* (1993), report using a scalarizing function, in the spirit of Wierzbicki (1980), as part of their procedure to reduce a large number of policy scenarios to a smaller set. In his paper of 1980, Wierzbicki tries to justify the belief that real decisions are made with the use of reference objectives or aspiration levels, rather than the opinion that single decisions are made by maximizing a utility or value function. He perceived the reference point to be neither an ultimate aspiration level nor merely a minimum necessary level of performance. Wierzbicki defines a scalarizing function to be optimized which, according to Stewart (1992), is really a "surrogate value function" that is "defined so as to give first preference to improving the worst underperformances relative to the reference point". A typical scalarizing function (to be minimized) is shown below,

$$\underset{\{1 \le i \le k\}}{MAX}\; w_i(z_i^* - z_i) + \varepsilon \sum_{i=1}^{k} w_i(z_i^* - z_i)$$

where z_i^* is some reference level for attribute (objective) i and the weights, w_i, would be proportional to the reciprocal of the range from nadir to ideal values.

The procedure is for the DM to continue modifying his or her expectations, as represented by the reference levels, until no further gains are perceived.

2.3 MCDM and Related Approaches Relevant to Analysing the Foreground Set

The methods discussed in the previous section apply in principle to any decision space, finite or infinite. The common assumption is that criteria are representable by quantifiable attributes, and this can be problematical when criteria are highly qualitative and/or represent group or political interests. In such cases value judgements cannot be expressed in terms of preferences between vectors of attributes. Value judgements can then only be expressed by direct comparisons of real alternatives. For example, in using MAUT, marginal utilities in terms of each criterion would be assessed by direct rankings of alternatives as in SMART (Simple Multi–Attribute Rating Technique) – see von Winterfeldt and Edwards (1986) or Goodwin and Wright (1991) for a discussion of this technique. This requires that, for purposes of analysis, the large set (decision space or Background Set as previously defined) be reduced to a smaller set, or the Foreground Set, as we will refer to it.

The process of reducing a large set of policy alternatives to a smaller set is not very well documented in the literature. Authors have tended to refer to this process (or processes similar to it) as "screening procedures". According to Walker (1986), two properties of good screening procedures are:

"(1) no very good alternatives will be missed, and

(2) the number of alternatives to be evaluated in impact assessment will be relatively small."

These guidelines or properties are all very good, but they do not provide specifics. The work of Miller (1956), however, suggests that 7 (plus or minus two) scenarios will be an ideal number on which a direct comparison could be made. Miller states:

> "...the span of absolute judgement and the span of immediate memory impose severe limitations on the amount of information that we are able to receive, process, and remember."

Human cognitive skills can therefore cope with at most 5 to 9 separate items of information at any one time. MCDM and related methodologies can be utilised in the process of selecting a short–list (or Foreground Set) consisting of such 5 to 9 policy scenarios. Two approaches, discussed in the previous section, are well–suited to the process of reducing a large set of alternatives to a short–list.

These are (i) the technique of Steuer (1986), termed the interactive weighted-sums method, which we discussed in section 2.2 (b), and (ii) the ideas of Wierzbicki (1980), which we discussed in section 2.2 (c). These two approaches were utilized (in combination) by Stewart *et al.* (1993), to select a Foreground Set of scenarios (see appendix C of Stewart *et al.* for a full description of the combined procedure). In our simulation study we have used a similar procedure to select a Foreground Set of alternatives, and we provide a full description of this procedure in section 3.5 of chapter 3.

In the remainder of this section, we will examine the overall process whereby a final solution set is extracted from the newly formed Foreground Set of alternatives. We will first look at which MCDM and related methods support group decision conferencing. Secondly, we will focus on how the various conflicting interest groups may set about seeking consensus solutions. Finally, we will examine which MCDM methods can be used to revise or review the Foreground Set by using the output from the consensus seeking process.

2.3.1 Group Decision Support

Group decision making and negotiation support are being studied more frequently in the literature due to the importance of the process and its applicability to modern day problem solving situations. The paper by Jelassi *et al.* (1990), for instance, distinguishes between 4 types of multi-person decision making situations: (i) individual decision making in a group setting, (ii) hierarchical or bureaucratic decision making, (iii) group decision making or one party decision making, and (iv) multi party decision making or negotiation. This paper provides an overview of formal models for group decision making and negotiation and places a special focus on those models that can be implemented in a computer based DSS. There is a growing realization in the literature that the use of modern microcomputers with their associated information technology, is becoming an absolute necessity for facilitating the process of group decision making.

Decision analysis should ideally be carried out within the group. The model, according to French (1989), should be developed and analyzed in the group environment. He further states that the members need to see, understand and be able to contribute in their own language throughout the modelling process. The power of the modern portable microcomputer and its graphical capabilities (together with overhead or video projection), means that decision analysis can be carried out almost anywhere. French continues to discuss the relationship and close interaction between the decision analysts or "facilitators", as they are becoming known, and the decision making groups. He also develops some further ideas for the "decision conferencing" format but admits that this format is evolving with experience. The reader is also referred to Goodwin and Wright (1991), wherein they discuss the role of decision analysis with special emphasis on decision conferences and problems involving the allocation of resources.

We will be focusing on some of the MCDM techniques that could be applied to the group decision making process in the light of the overall process we have previously described, i.e. extracting a solution set from the Foreground Set of policy scenarios.

(a) Value Functions

We have seen from section 2.2 (a) that the value function (utility based) approach is enormously helpful for the DM to gain further insight into his or her preference structure. Keeney and Raiffa (1976), discuss some of the uses of the utility function approach or that of multiattribute utility analysis in aiding group decision making.

In French (1989), a paper by Belton is presented that describes the use of a simple multiple criteria model as a decision aid to a large service company. The paper describes how the model aided the company to select a computer system from a short-list of alternatives. One of the main features of the model that was appealing to the members of the decision making group, and was also of particular benefit to the process as a whole, is the graphical representation of the results. The scoring of the various criteria was presented on a 0–100 scale with the worst and best options constrained to 0 and 100 respectively. Some members of the group were somewhat uneasy with the use of such a scale, but Belton feels that the feelings of unease are generally overcome with familiarity.

Stewart *et al.* (1993), strongly recommend the VISA (Visual Interactive Sensitivity Analysis) computer package for use in any implementation of scenario based planning procedures. This package is developed by Belton and Vickers (1989), and Stewart *et al.* demonstrate how the package could be used to facilitate consensus seeking both between individual members of a group (i.e. a "within-interest" group phase) as well as between different decision making or interest groups (i.e. a "between–interest" group phase). The 0–100 scale is termed a "thermometer scale". At the within–group phase of an analysis it could be used to represent and evaluate the scores (of the alternatives) for the different attributes or criteria of the particular group. At the between–group phase it could in turn be used to evaluate the different alternatives of the short–list or Foreground Set, the alternatives now having aggregated scores for the criteria of the various groups, in order to make a final selection.

Belton and Vickers (1990), state that the approach on which VISA is based is best suited to "the problem of choosing a preferred alternative from a set of well defined alternatives, or to indicate a preference ordering over such a set of alternatives". The indication of a preference ordering over a set of well defined alternatives, will be of value to the process of seeking a consensus solution as well as the process of revising or re–evaluating the Foreground Set of policy scenarios. The process of choosing a preferred alternative from a set of well defined alternatives, is completed by using the model structure below, where the model determines an overall value, V_i, for each alternative i.

$$V_i = \sum_{j=1}^{k} w_j v_{ij}$$

where v_{ij} = evaluation (or score) of alternative i on criterion j, and
w_j = relative importance (weight) of criterion j.

Before choosing the best alternative(s), a thorough sensitivity analysis has to be conducted, especially on the criteria weights. The authors warn that the results provided by this model, *vis–à–vis* the overall scores of the alternatives, should not be "unquestioningly accepted as the answer". If the results of the analysis is that two or more alternatives perform similarly well overall, then the user should not blindly accept the resulting ranking, but should explore the results in greater depth. After all, the solution is a set, and this set is not necessarily limited to one alternative only. The richness of the graphical presentation of alternative profiles and sensitivity analysis, together with the interactive analysis provided in VISA, is a good environment for the further exploration of the results.

(b) Goal Programming in Interactive Mode

The goal programming approach, in interactive mode, has been found to be a very transparent procedure. Stewart and Brent (1988), report that the IMGP procedure of Spronk (1981) would be "valuable in a group decision aiding context". They also report that a number of users, from widely differing backgrounds (marine scientists, economists and industry management), experimented with a prototype IMGP system that the authors had developed. All the users were able to reach compromise solutions that satisfied their decision goals. Furthermore, the extent to which the solutions obtained by the widely differing interest groups tended to converge, provided further support for using the IMGP as part of their proposed DSS.

Khorramshahgol and Steiner (1988), recognize that a major drawback of the goal programming procedure is that the DM must specify his or her goals and priorities *a priori*. The authors suggest using the Delphi method in an attempt to overcome this problem. DMs will therefore structure their objectives prior to goal programming as follows: (i) they will identify the goals, (ii) they will determine priorities among the goals, and (iii) they will establish a target level for each goal. The Delphi method is a systematic procedure to obtain a consensus from a group of participants that makes use of written responses. This will avoid the problem experienced in face–to–face meetings in which a few influential members can dominate the process. It allows for eliciting expert opinion in an iterative process. Because the Delphi method is therefore an iterative process and it preserves the anonymity of the group members, Khorramshahgol and Steiner feel that it holds decided advantages over other techniques. The reader is referred to Delbecq *et al.* (1975), for a further description of the Delphi method. von Winterfeldt and Edwards (1986), present a brief discussion on the Delphi method. They also cite references wherein the Delphi method, the Nominal Group Technique and free–form discussion have been compared experimentally for the purpose of probability estimation.

Group decision making, negotiation and consensus seeking between conflicting interests are processes that cannot be restricted and rigidly administered by means of pre–specified algorithms. Algorithms can and should, however, provide a framework for these processes to be carried out within. The role that computer based DSS can play in facilitating this process is a crucial one. Empirical evidence does, however, still need to be acquired to substantiate some of the computer based DSS that have been proposed in the literature.

2.3.2 Seeking Consensus Solutions

The process of seeking a consensus solution should be iterative in nature, so that if a consensus cannot be reached, the Foreground Set can be revised before we attempt to seek a further solution. The MCDM approaches that could utilize the outputs from such a consensus seeking process are discussed in section 2.3.3. In this section we will be concentrating on applying MCDM and related approaches to seeking consensus between interests and/or groups, when the preferences of each interest or group (over the Foreground Set) have been expressed as thermometer scales as described in section 2.3.1 (a).

(a) Value Functions and Goal Programming in Interactive Mode

From our previous discussions in sections 2.2 (b) and 2.2 (c) we commented that both the value function and goal programming approaches respectively, in interactive mode were favoured to the same approaches when not used in interactive mode. The interactive nature of these approaches would allow the DM to re-examine the final solution set.

For the interactive utility based procedures, we have previously seen that the methods are well suited to reducing a large number of alternatives to a smaller set. Once the smaller set has been established, the procedure could be re-applied in order to extract a solution set. The ideas of Steuer as well as those of Zionts and co-workers, as previously discussed in section 2.2 (b), could be used for this purpose, although, according to Stewart (1992), they are better suited to the process of reducing the large set of policy scenarios to a Foreground Set. The difference would be that they will now be applied across the various conflicting interests with the questions being posed to the various groups.

Under the broad category of interactive goal programming methods, we have stated in section 2.3.1 (b) that the IMGP procedure of Spronk (1981), is according to Stewart and Brent (1988), valuable in a group decision aiding context. Stewart (1988), however, also reported that the solution form of producing a short-list of alternatives by this method was preferred (by DMs) to producing a single solution. This would allow the DMs to re-assess the solution set (including the re-assessment of their goals) before they have to make a final decision, and would therefore make the IMGP method suitable for the purpose of extracting a solution(s) from the Foreground Set of policy scenarios. The manner by which the solution set is extracted from our Foreground Set, would be by applying goal programming to the interest group thermometer scales. The goal programming approach that was used in our simulation study is reported in section 3.7.3 of chapter 3, under the heading of Maximum/Minimum (Max/Min).

The Compromise Programming (CP) method, as discussed by Duckstein *et al.* (1993), is a distance-based technique that was developed by Zeleny (1973). It is designed to identify so-called compromise solutions, that are determined to be the closest solutions, by some distance measure, to an ideal (but infeasible) solution. A distance measure that can be used in CP and one of the most frequently used measures of closeness, is a family of weighted L_p metrics, defined as follows:

$$L_p = \left[\sum_{i=1}^{k} w_i^p \left| \frac{z_i - z_i^*}{range\ (criterion\ i)} \right|^p \right]^{1/p}$$

where z_i^* = best (optimal) value for criterion i,

w_i = weight for criterion i, and

$p \in [1; \infty]$.

The compromise solution, a feasible solution that is as close as possible to the ideal in terms of the chosen metric (with given p and w_i), then results from the following optimization problem:

$$\underset{\{alternatives\}}{Min} \quad L_p$$

for all alternatives in the feasible space.

For a given set of weights $\{w_i\}$ and for all p, the set of compromise solutions, called the compromise set, is obtained. Usually, only three points of the compromise set are calculated, *viz.* the extremes for $p = 1$, $p = \infty$ and an intermediate point for p, say, $p = 2$. It is worth to note that the CP method, for $1 \leq p < \infty$, always produces a non–dominated solution. For $p = \infty$ the uniqueness of the solution, which is the same as the Tchebycheff solution, is not guaranteed. Stewart (1992) suggests that this can be avoided by ensuring that the set of alternatives is Pareto optimal at the outset. When this is, however, not feasible or practical, Stewart goes further to suggest a modification of the distance measure. The reader is referred to Stewart (1992), for more information on this modified distance measure.

A positive factor of the CP method is that it produces a compromise set of solutions. The DMs, however, still have to specify the weights (w_i) and the distance function(s) (i.e. choose a value for p) to be used. For each point in the compromise set (i.e. a value for p), a solution set can be extracted according to the rankings produced by the method. This, ideally, can be repeated for several points in the compromise set and a comparison can be made between the various solution sets that are extracted. This will allow the DMs to justify their recommendation(s), since they would have gained further insight into the problem by comparing the outcomes produced by the various distance functions.

(b) Outranking Approach

The concept of outranking relations between alternatives was born out of difficulties encountered with diverse problems according to Roy (1990). The outranking philosophy as such, was developed by Roy and co–workers in Paris and enjoys popularity in Europe. This philosophy is implemented in the various versions of the ELECTRE method, ELECTRE being a French acronym for *ELimination Et Choisis Translation REalité* (or elimination and choice algorithm). For the various versions of ELECTRE, the reader is referred to Roy (1990).

Outranking between alternatives a and b, say, in the sense of ELECTRE is based on two requirements:

(1) alternative a has to be better than b in a sufficient number of sufficiently important criteria, and

(2) alternative a must not be much worse than b in the remaining criteria.

The first requirement is embodied in a concordance index C_{ab}, and the second in a discordance index D_{ab}. Therefore, for any pair of decision alternatives (a and b, say, with attribute vectors $\mathbf{z}^a$ and $\mathbf{z}^b$), two quantities are defined as follows. The first quantity is,

$$\textit{Concordance:} \qquad C_{ab} = \sum_{i:z_i^a > z_i^b} \{w_i\}$$

where the w_i are i attribute weights (usually normalized to sum to unity), and where in the case of ties, half the weight is counted.

This definition can be interpreted as the sum of the weights of criteria or attributes for which alternative a is better than alternative b. These weights are not seen in tradeoff terms, but are rather relative voting weights accorded to the respective criteria. The second quantity is,

$$\textit{Discordance:} \qquad D_{ab} = \underset{i}{Max} \left[\frac{(z_i^b - z_i^a)}{S_i} \right]$$

$$= Max \; \{ \; \underset{i}{Max} \left[\frac{(z_i^b - z_i^a)}{S_i} \right]; \; 0 \; \}$$

where S_i is some suitable scaling factor, eg. the range of attribute values for attribute i, $\mathrm{Max}_{x \in X}[z_i] - \mathrm{Min}_{x \in X}[z_i]$.

The interpretation given to this definition is the largest scaled difference in favour of alternative b, taken over all criteria or attributes.

Conventionally, the concordances and discordances are displayed as nxn matrices, where n is the number of alternatives. If alternative a dominates alternative b then $C_{ab} = 1$ and $D_{ab} \leq 0$, but in general for two non-dominated solutions $C_{ab} < 1$ and $D_{ab} > 0$. In general, however, the principle states that alternative a "*outranks*" alternative b if there is insufficient reason to conclude that b is better than a. Thus if a dominates b then a outranks b, but b does not outrank a. However, we could have b outranking a at the same time as a outranks b.

The ELECTRE I method uses the concept of outranking in a stronger sense than dominance. It combines the concordance and discordance indices by comparing each index to respective threshold levels, say, c^* and d^* respectively. Alternative a is then said to outrank b if:

$$C_{ab} \geq c^*$$

and

$$D_{ab} \leq d^*.$$

Furthermore, if $c^* = 1$ or $d^* = 0$, then a outranks b if and only if a dominates b. Therefore to strengthen the idea we would need to use $c^* < 1$ and/or $d^* > 0$. These two index value comparisons are then used to obtain an outranking matrix and the elements of this matrix say, r_{ab}, can be described as follows:

$$r_{ab} = \begin{cases} 1 \ \ if \ C_{ab} \geq c^* \ \ and \ \ D_{ab} \leq d^* \\ 0 \ \ otherwise \end{cases}$$

This outranking matrix embodies a relation R, that can be analyzed in several ways. From this outranking matrix, a set of alternatives can be formed in such a way that they are not outranked by other alternatives. The definition of this set is similar to that of efficient alternatives (in the *Pareto* sense) and is the following:

$$E(R) = \{a \mid r_{ba} = 0, \text{ for every alternative } b\}$$

The approach of analysing the outranking matrix is that of obtaining a partial rank ordering of the alternatives. In simplified terms, the method works if there are neither too many nor too few outranking relationships. Another way of displaying the outranking relationships is using a graphical representation, with nodes representing alternatives, and directed arcs representing outrankings. These outranking graphs, in particular, help DMs focus their attention on critical issues and gain further insight into their own preference structures.

The valuable insights gained from the ELECTRE I method can only be achieved when the number of alternatives under consideration are relatively small. Stewart (1992), suggests a number of 6 or less, although this is a subjective opinion based on the author's experiences. It is quite plausible that a larger number of alternatives may also produce valuable insights for the DM. What is obvious though, is that the level of insight attained would be directly linked to the number of alternatives being considered. The larger the number of alternatives being considered, the smaller the amount (or level) of insight that will be attained.

Furthermore, it is important how these insights may be used to discard certain alternatives from further consideration. The outranking matrix that produces the partial rankings of alternatives, together with the graphical representation of these outranking relationships, will determine which alternatives should be retained and which should be discarded. There is, however, no fixed rule that states how this information should be used if one were to stop at this point. Each problem situation is unique and DMs will use the information as an aid or as a basis upon which to make further decisions.

If one was to continue with the outranking approach, then the ELECTRE II method, using the preference graphs as input, can provide a complete ordering of the alternatives. Gershon *et al.* (1982), suggest that the ELECTRE I and II methods could be used to obtain "a complete ordering among these systems based upon quantifiable and nonquantifiable ordinal criteria". The systems they refer to are alternative river basin (management) strategies that involve qualitative ordinal criteria. They further propose that ELECTRE I be applied for the purposes of screening where the number of alternative systems become excessively large, and ELECTRE II be applied to rank the remaining systems.

Two preference graphs must be generated by ELECTRE I for use as input to the ELECTRE II procedure. These graphs will represent the strong and weak preference structures of the DM. The strong preference graph results from the use of stringent threshold values. This means that the DM will use a high level of concordance (c^*) and a low level of discordance (d^*). For the weak preference graph, the DM will relax his or her threshold values by lowering the level of concordance and raising the level of discordance. The relaxed threshold values represent lower bounds on system performance that the DM is willing to accept. The ELECTRE II approach requires two separate rankings, referred to as the forward and reverse rankings.

The first step in the forward ranking procedure is to obtain a set, named C, the elements of which are all nodes (alternatives) in the strong preference graph that are not outranked by any other alternatives. Next, the alternatives in set C that are not outranked in the weak preference graph are identified and defined as set A. The elements of set A are assigned the rank of one. The next step consists of reducing the weak and strong preference graphs by eliminating all alternatives that are part of set A and all arcs (outranking relationships) that originate from these alternatives. The reduced strong preference graph is re-examined and all remaining alternatives that are not outranked by any other alternatives now comprise a new set C. The iterative procedure is repeated with the elements of the new set A being assigned a rank two. This continues until all alternatives have been eliminated from the weak and strong preference graphs and thereby have a ranking assigned to them.

The first step of the reverse ranking procedure is to reverse the direction of all the arcs (outranking relationships) in both the weak and strong preference graphs. This reversal means that a high concordance relationship now becomes a low concordance, and a low discordance relationship now becomes a high discordance. The remaining steps are identical to those of the forward ranking procedure with one difference: the alternative that is ranked last is ranked first and the remaining alternatives are ranked in reverse order. This re-establishes the correct direction of the ranking process.

Upon completion of both the forward and reverse ranking procedures, an average ranking of the two is taken for each alternative. The final stage of ELECTRE II is to order alternatives with respect to their average rankings and thereby establish a complete ranking of the alternatives.

The ELECTRE II method, as a procedure on its own, does not allow for an interactive re-evaluation of the final solution set that is produced. It produces a strict and complete ranking of all the alternatives in the Foreground Set. This is perhaps advantageous in certain situations but can most certainly be disadvantageous in other settings. A better approach would be to use both ELECTRE I and II in the process of extracting a final solution set. By doing so, the DMs may be able to combine the information they gain from the partial rankings of ELECTRE I, with that of the strict rankings of ELECTRE II. This would allow them to at least further justify any conclusions that are reached, before any further recommendations are made.

2.3.3 Revising the Foreground Set

If the initial Foreground Set is insufficient to allow for the realization of aspirations of the interest groups, then the consensus solution will not have a high level of support from all the interests concerned. The Foreground Set will therefore have to be revised in light of the outputs obtained from the process of seeking a consensus solution. The scenario based planning procedure of Stewart *et al.* (1993), as previously outlined in figure 2.1, provides a schematic indication of how this might be achieved. Stewart *et al.* quote Bui's (1987) description of the essence of the consensus seeking process as follows:

> "In effect, when a decision maker first attempts to establish an order of preferences, his analysis often results in a ranking of the alternatives ...He would then logically choose the alternative that is ranked first in the vector of preferences. However, unless the chosen alternative obviously outranks its counterparts, there is no reason why the next ranked alternative could not be considered as a comparatively acceptable solution."

When there is more than one DM involved, this process of seeking consensus (as portrayed by revising the Foreground Set) becomes especially important. This does not mean to say that the process is irrelevant when only one DM is involved, since this single DM may have to justify his or her ultimate decisions to a third party. The process of consensus seeking adds further support in favour of the ultimate solution(s) that the single DM may present. The process can be accommodated by certain MCDM methods. It is, however, important to distinguish between problems involving single and multiple DMs. We discuss the MCDM methods under their broad headings below, and in this section a single DM is assumed. We can think of the single DM as a facilitator to the various conflicting interest groups. The appropriate MCDM methods that support multiple DMs in a group decision making process were previously discussed in section 2.3.1 of this chapter.

(a) Value Functions in Interactive Mode

We have noted in section 2.2 that value functions, more so in an interactive mode, induce a learning effect for the DM. The DM may then use these insights gained since they enable the DM to better understand his or her own preference structure.

Stewart *et al.* (1993), report that the ideas of Korhonen, Wallenius and Zionts (1984), which have been presented earlier in this chapter, can be used to facilitate the process of revising the Foreground Set by using the partial orderings of the alternatives. The Korhonen, Wallenius and Zionts method uses the DM's pairwise judgements to generate certain convex polyhedral cones that may eliminate alternatives. All alternatives (or solutions) that are dominated by these cones may be eliminated and need never be presented to the DM. The reader is referred to Stewart *et al.* (1993), appendix C, for a discussion on further ideas (*cf.* ideas of Wierzbicki in a goal programming context) that attempt to implement MCDM methodologies in order to accommodate the process of consensus seeking in the Foreground Set.

(b) Goal Programming in Interactive Mode

The IMGP method of Spronk (1981), presented in section 2.2 (c), can also facilitate the consensus seeking process. The fact that the DM can adjust his or her goal levels interactively (specifically the nadir values), indicates that the output from the process of seeking a consensus solution can be used by the DM to review his or her goal levels. This process of reviewing or resetting the goal levels, in light of further insights that may have been gained, will lead to a renewed evaluation of the Foreground Set of policy scenarios.

The fact that the DM has to specify overall goal levels and (penalty) weights both subjectively and *a priori* is a criticism that is levelled at the goal programming approach. The IMGP procedure does address the issue of adjusting the goal levels interactively, although the original intervals for these goals (i.e. the ideal and nadir values) still have to be specified when the large set of scenarios are being considered. However, the pre-specification of penalty weights still remains a valid criticism. These weights could possibly be re-examined by the DM once the Foreground Set has been established or once some additional insight has been achieved during the process of seeking a consensus solution. Once these weights have been reviewed, the Foreground Set of policy scenarios may then be revised in order to reflect the new set of penalty weights, and also the revised goal levels.

CHAPTER 3: DESCRIPTION OF THE APPROACH ADOPTED FOR THE STUDY

3.1 Overview of the Simulation Approach

In order to accommodate the objectives of this study, as outlined in section 1.4 of chapter 1, we have chosen to adopt a simulation approach. This approach has both decided advantages and disadvantages attached to it. These are briefly discussed in section 1.5 of chapter 1, and we will deal further with some of the limitations encountered, once we report on the results of the study in chapters 4 and 5.

As stated in section 1.4 of chapter 1, we will focus in particular on two areas of the procedure for scenario based policy planning, as developed by Stewart *et al.* (1993), (we have previously provided their full procedure diagrammatically in figure 2.1 of chapter 2). The areas we will attempt to simulate are:

(1) reducing a large set of policy scenarios (analogous to our Background Set) to a more manageable smaller set (analogous to our Foreground Set), and

(2) further refining the smaller set to extract a solution set from it.

The simulation study algorithm we have used is summarized in figure 3.1 on the following page, for a particular iteration (decision event), and described in detail in the remaining sections of this chapter. For a particular simulation run, 100 iterations were carried out before the final statistical analysis was conducted. There are, however, a few details that need to be mentioned before we continue.

The first is that the algorithm was applied separately to two data sets, both serving as the Background Set of policy scenarios. These two data sets are described in sections 3.2 and 3.3 in more detail. We will, however, conduct our analysis, in terms of simulating possible real life decision events, separately for each of the two data sets. The reasons for doing so are the following:

(1) We used table B1 in Appendix B of Stewart *et al.* (1993), to serve as the first data set. The full table is provided in section 3.2 below and the data, although being hypothetical, represents realistic scenarios for the development of the Sabie–Sand river catchment area in the Eastern Transvaal region of South Africa. The data has already been used by Stewart

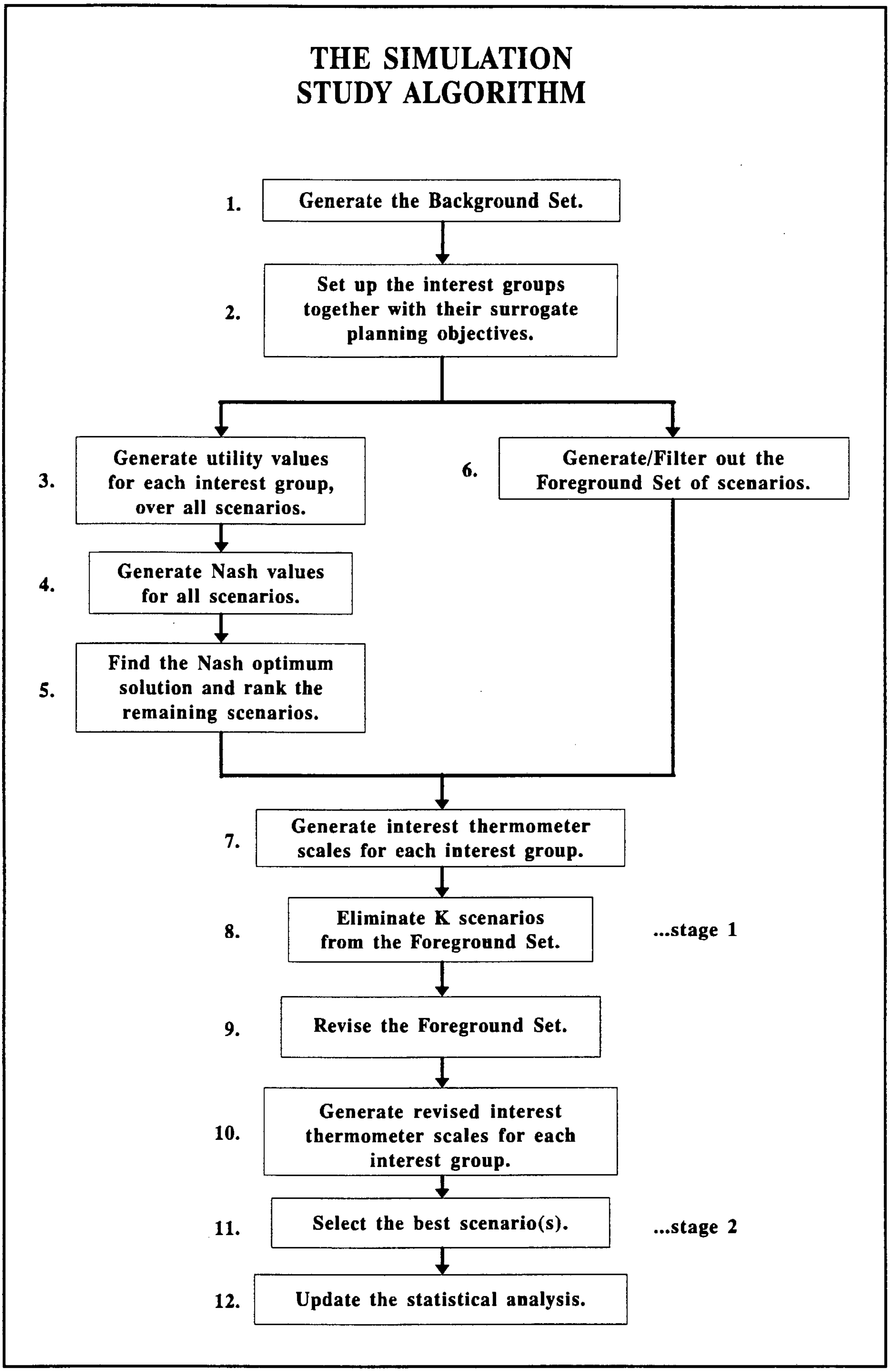

Figure 3.1: A schematic representation of the simulation study algorithm

et al. to illustrate the various MCDM approaches (that the authors considered for their own purposes), as well as some of the strengths and weaknesses of the methods. Therefore, the data contained in this table would be used as a *"preliminary"* type of investigation into the various MCDM methods that we will be considering in this study.

(2) The second data set we used was randomly generated using an adapted experimental design procedure. The data set also changed from one iteration to the next. Further details about the experimental design procedure are provided in section 3.3. Results obtained from this data set would therefore hopefully serve as a confirmation of the findings from the first data set. New findings are also expected and these will be reported in chapter 5.

(3) External factors, that may or may not affect the results yielded by the various methods, could also be examined by using the second data set. This data set more closely provides an *"aggregate measure"* of numerous decision events since the data changes for every iteration of a particular simulation run. (The effects of changing external factors, such as the number of interest groups, are reported separately when we report the results for the second data set in chapter 5.)

The second detail to be mentioned is that we have labelled two steps of the simulation algorithm, *viz.* *"stage 1"* and *"stage 2"*. MCDM and related methods are applied at these two steps of the simulation procedure. These steps or stages represent part of the process of revising the newly formed Foreground Set of policy scenarios (stage 1), and the process of extracting a solution set of policy scenarios (stage 2). The steps were labelled so that the MCDM methods, used in a certain simulation run, could be identified at (or associated with) the particular step of the simulation procedure, at which they were being used.

For the remainder of this chapter we will describe in more detail the two data sets (sections 3.2 and 3.3 respectively). This will reflect the first two steps (i.e. 1 and 2 as labelled) of the simulation algorithm illustrated in figure 3.1. The remaining steps of the algorithm are described in this chapter as follows: steps 3, 4, and 5 in section 3.4; step 6 in section 3.5; step 7 in section 3.6; steps 8 and 9 in section 3.7; steps 10 and 11 in section 3.8; and step 12 in section 3.9.

3.2 Summary of the First Data Set

The first data set, provided in table 3.1 on the following page, is taken from table B1, Appendix B, of Stewart *et al.* (1993). Table B1 was used by Stewart *et al.*, to illustrate the various MCDM approaches the authors reported on in their study, including the strengths and weaknesses of these methods. The authors also hoped that the hypothetical data for the particular region could serve as a demonstration project.

Columns 2 to 5 of table 3.1 comprise the *policy elements* and columns 6 to 9 the *derived attributes* or *criteria* for this data set. Together, the values for columns 2 to 9 will form the 8 *attributes* by which the 20 policy scenarios are evaluated for the first data set. This data set represents simplified and hypothetical policy scenarios for a region such as the Sabie–Sand River system in the Eastern Transvaal. The reader is, however, referred to Appendix B of Stewart *et al.* (1993), for further background information on this data set.

For the purposes of our study, the data in table 3.1 will be read as follows: z_{ij}, for *decision alternatives* or *policy scenarios* $i = 1, ..., 20$, and *attributes* $j = 1, ..., 8$. Furthermore, there are $p = 4$ *interest groups* defined as follows:

(1) the *forestry* interest group,
(2) the *irrigation* interest group,
(3) the *rural* communities interest group, and
(4) the *conservation* interest group.

For each interest group, a particular set of attributes that apply to the interest group has been selected from the total of 8. These sets, as they apply to the specific interest group, are defined below. Accompanying each attribute will be its *direction of preference* (i.e. maximizing or minimizing), that is unique to the particular interest group, and this forms the $q = 11$ *surrogate planning objectives* for this data set.

(1) Forestry interest group:
 (a) (column 2) Percentage change in afforestation, relative to *status quo*, permitted or enforced, with a feasible range of options from −3% (i.e. a 3% reduction) to +5% – *maximize*;

TABLE 3.1: Data Set No. 1

(Simplified and Hypothetical Policy Scenarios for a region such as the Sabie)

Policy Scenarios	Attributes							
	% Change in Forestry	% Cut in Irrigation	Dam size (% of Max.)	% Rural Populn. Served	Cost (Rm)	% Change Annual Flow	% Change Low Flows	% Change Peak Flows
1	-3	20	60	30	450	20	50	-5
2	-3	35	80	40	575	17	65	-13
3	-3	50	60	40	500	15	65	-5
4	-3	50	100	50	650	15	80	-20
5	-1.5	20	60	30	450	7	35	-8
6	-1.5	35	80	40	575	5	50	-15
7	-1.5	50	60	40	500	2	50	-8
8	-1.5	50	100	50	650	2	65	-23
9	0	20	60	30	450	-5	20	-10
10	0	35	80	50	625	-1	35	-14
11	0	50	60	40	500	-10	35	-10
12	0	50	100	50	650	-10	50	-25
13	+2.5	20	60	30	450	-26	-5	-14
14	+2.5	35	80	50	625	-22	10	-18
15	+2.5	50	60	40	500	-31	10	-14
16	+2.5	50	100	60	700	-24	25	-26
17	+5	20	60	40	500	-40	-30	-15
18	+5	35	80	50	625	-43	-15	-23
19	+5	50	60	30	450	-58	-15	-22
20	+5	50	100	60	700	-45	0	-30

(b) (column 4) The sizing of the dam to be constructed at a predetermined site, expressed as a percentage of the optimal size from engineering or economic considerations, which may range between 60% and 100% - *maximize*;

(c) (column 6) Total investment costs in millions of Rand - *minimize*.

(2) Irrigation interest group:

(a) (column 3) Percentage reduction in current levels of irrigation for agriculture in the area, with permissible values between 20% and 50% of current use - *minimize*;

(b) (column 5) Percentage of the rural population to be provided with standpipes within 100 metres of their dwellings, with values ranging between 30% and 60% - *maximize*.

(3) Rural communities interest group:

(a) (column 5) Percentage of the rural population to be provided with standpipes within 100 metres of their dwellings, with values ranging between 30% and 60% - *maximize*;

(b) (column 8) Percentage change (from *status quo* conditions) in the minimum (low) flow during the year - *maximize*;

(c) (column 9) Percentage change (from *status quo* conditions) in the peak flow during the year - *minimize*.

(4) Conservation interest group:

(a) (column 7) Percentage change (from *status quo* conditions) in total annual flow - *maximize*;

(b) (column 8) Percentage change (from *status quo* conditions) in the minimum (low) flow during the year - *maximize*;

(c) (column 9) Percentage change (from *status quo* conditions) in the peak flow during the year - *maximize*.

For the simulation process on this data set, the sets of attributes together with their directions of preference, as they apply to the 4 interest groups, will remain unchanged from one iteration to the next for a particular simulation run. What was varied (between iterations), is the sets of utility functions that apply to each attribute of a particular interest group.

3.3 Summary of the Second Data Set

The second data set or randomly generated data set, as it will sometimes be referred to, was formed in such a way that it would closely resemble the first data set. The main reason for this is that we wanted to test whether the results obtained from the first data set would be repeated in the second. Furthermore, we were also interested in obtaining any new results as well as exploring the effects of external factors on the decision making process. These factors include the effects of the number of interest groups, the effects of the number of policy elements and the effects of the number of derived attributes on the results.

To start, we used $p = 4$ interest groups, $m = 4$ policy elements generated by means of an experimental design procedure and $n = 4$ derived attributes generated from these policy elements. In order to simulate real life decision events, we re-generated the derived attributes from one iteration to the next. To be accurate, we should therefore be referring to this data set as "*randomly generated data sets*" and not just, by implication, a "single" randomly generated data set. The latter is used, however, for ease of distinction. The manner in which interest groups were assigned attributes (z_i, for the i-th attribute), consisting of combinations of policy elements (x_i) and derived attributes (y_j), as well as how this assignment varied from one iteration to the next, will be discussed later in this section.

The experimental design procedure used to generate the scenarios reflecting the policy elements, is similar to the one used by Stewart *et al.* (1993), in their appendix C. The authors use this procedure to generate their Background Set of scenarios, as defined by combinations of (m) policy elements. An argument is presented by the authors for not using the full 2^m factorial design, for the m policy elements. This argument is based mainly on practical considerations that need to be taken for large values of m. With m increasing in size, the full 2^m factorial design will require an even quicker increasing amount of intensive computer processing. Instead, Stewart *et al.*, propose using a particular version of the factorial design procedure that is aimed specifically at maximizing the spread of scenarios which are to be considered.

The full design procedure will be headed under a "*partial factorial design set*" and an "*extended centre point set*". The latter refers to an "extended centre point" that ensures an additional richness in the centre ($x_i = ½$) of the design set, i.e. with values for x_i between ¼ and ¾. x_i is defined as the level chosen for the i-th policy element, on a standardized scale on which the minimum value is 0 and the maximum value is 1. In some cases, x_i may only be able to take on the values 0 or 1; for example, one policy element may refer to whether or not a particular dam is built. In this case x_i = 0 means no dam is built and $x_i = 1$ means that a dam is built.

The design procedure proposed by Stewart *et al.*, is the following:

Partial Factorial Design Set:

For $m \leq 4$, use the complete 2^m factorial design (see example in table 3.2 on the following page).

For $m > 4$, the following design, consisting of $(2+m+m^2)$ scenarios is proposed:

One scenario defined by $x_i = 0$ for all policy elements i

m scenarios, in each of which $x_j = 1$ for a particular policy element j, and $x_i = 0$ for all $i \neq j$

$m(m-1)/2$ scenarios, in each of which $x_j = x_l = 1$ for a particular pair of policy elements j and l, and $x_i = 0$ for all $i \neq j,l$

$m(m-1)/2$ scenarios, in each of which $x_j = x_l = 0$ for a particular pair of policy elements j and l, and $x_i = 1$ for all $i \neq j,l$

m scenarios, in each of which $x_j = 0$ for a particular policy element j, and $x_i = 1$ for all $i \neq j$

One scenario defined by $x_i = 1$ for all policy elements i

This design allows for the estimation of all main effects of each policy element, as well as all first-order interaction effects.

Extended Centre Point Set: (For continuous policy elements only)

The following (1+2m) scenarios are proposed:

TABLE 3.2: Illustrating the proposed experimental design procedure for m = 4 elements

Policy Scenarios	Policy Elements			
	x_1	x_2	x_3	x_4
1	0	0	0	0
2	1	0	0	0
3	0	1	0	0
4	0	0	1	0
5	0	0	0	1
6	1	1	0	0
7	1	0	1	0
8	1	0	0	1
9	0	1	1	0
10	0	1	0	1
11	0	0	1	1
12	1	1	1	0
13	1	1	0	1
14	1	0	1	1
15	0	1	1	1
16	1	1	1	1
17	½	½	½	½
18	¼	½	½	½
19	½	¼	½	½
20	½	½	¼	½
21	½	½	½	¼
22	¾	½	½	½
23	½	¾	½	½
24	½	½	¾	½
25	½	½	½	¾

One scenario defined by $x_i = ½$ for all policy elements i

m scenarios, in each of which $x_j = ¼$ for a particular policy element j, and $x_i = ½$ for all $i \neq j$

m scenarios, in each of which $x_j = ¾$ for a particular policy element j, and $x_i = ½$ for all $i \neq j$

This design allows for the estimation of the additional richness of the centre point set. As an example of the above design procedure, we listed the $2^4 + (1+2(4))$ policy scenarios generated by the procedure in table 3.2 on the previous page, for m = 4 policy elements. All the policy elements in this example are continuous variables.

The values shown in table 3.2 are all expressed in standardized form, where, as stated before, $x_i = 1$ corresponds to the maximum value (say, z_i^{max} for attribute i), and $x_i = 0$ to the minimum value (say, z_i^{min}), for the i-th policy element. The standardized values can be converted back into their natural units for each policy element. The relationship between the x_i's and the z_i's, that can be used to carry out this conversion, is shown in the equation below:

$$x_i = \frac{(z_i - z_i^{\min})}{(z_i^{\max} - z_i^{\min})}$$

By using this design procedure, the number of policy scenarios do not increase (as the number of policy elements increase) as rapidly as they would for the full 2^m factorial design. This design procedure would therefore make the data set computationally easier to work with, when there is an increasing number of policy elements used in the study.

The derived attributes (y_j), are generated for this data set as a function of the policy elements (x_i), or $\mathbf{y} = f(x_1, x_2, \ldots x_m)$. We calculated the j-th derived attribute (for j = 1 to n) as follows:

$$y_j = \sum_{i=1}^{r} \beta_{jr} (f_i)^2$$

for coefficients, β_{jr}, random on the Uniform [−1; +1] distribution, and

where we define r (r < m) factors, f_i, in the sense of multivariate factor analysis, as follows:

$$f_i = A^{(i)} \boldsymbol{x}$$

$$= \sum_{l=1}^{m} \alpha_{il} x_l$$

for coefficients, α_{il}, also random on the Uniform [−1; +1] distribution.

We used r = 3 factors when generating the derived attributes for the second data set, and decided to keep this constant value of r for all the simulation runs.

Furthermore, for a particular iteration of a simulation run, each interest group was assigned a random number of attributes. This random number changed from one iteration to the next. The attributes (z_j), are made up of a randomly chosen number of policy elements (x_j), as well as a random number of derived attributes (y_j). The policy elements remained the same for all iterations, but the derived attributes changed from one iteration to the next. The attributes are then selected in order to apply to a specific interest group, for the particular iteration. Overlapping of attributes was allowed in a particular iteration, but of course each policy element or derived attribute, will also have a randomly chosen direction of preference associated with it. This will be unique to a particular interest group, in a particular iteration. We allowed each interest group to have the same number of policy elements and derived attributes applicable to it. However, which policy elements and which derived attributes, together with their directions of preference, remained different for each of the interest groups.

To sum up, we have changed the policy elements and derived attributes, as well as varied the degree of overlap of both policy elements, derived attributes and also directions of preference, from one iteration to the next. The external factors that may affect our results could be varied using this randomly generated data set. These started out at p = 4 interest groups, m = 4 policy elements and n = 4 derived attributes for this data set. These values were subsequently changed in further simulation runs and any effects that resulted from such changes, will be reported in chapter 5.

3.4 Utility Values and the Nash Optimum

Once we had established the Background Set of policy scenarios, we were able to start simulating the DMs' responses to the various MCDM approaches. First we needed to generate a value or utility function for each attribute of each interest group. We therefore simulated a preference structure that is assumed to satisfy the axioms of MAUT including *preferential independence*. Carrying out this approach under the assumption of preferential independence, (as noted in section 2.2 (a) of chapter 2), requires that the relative strengths of preference between scenarios differing on one criterion only, are independent of levels of achievement on the other criteria.

For a particular interest group, the MAUT model would require two steps. The first is the specification of *marginal value functions* (shown as $v_{Fj}(z_{Fj}; R_{Fj})$ below – "F_j" for the forestry interest group of the first data set, say), for each of the attributes (j) of the interest group. The second step would require the specification of *importance weights* (shown as w_j below). The utility function for, say, the forestry interest group of the first data set, could then be represented as follows:

$$V_F(z_F) = \sum_{j \text{ attributes of interest}} w_j v_{F_j}(z_{F_j}; R_{F_j})$$

The utility value provided by the utility function $V_F(z_F)$, will therefore be calculated for each policy scenario in the Background Set. Furthermore, for the forestry interest group of the first data set, used as an example again, we would have j = 3 attributes, with:

w_1 being the weight for the percentage change in afforestation attribute,
w_2 being the weight for the dam size (as a percentage of the optimal size) attribute, and
w_3 being the weight for the total investment costs (millions of rand) attribute.

The weights (rescaled to sum to 100) used in the first data set of this study, together with the interest group to which they apply, are the following:

Forestry: $w_1 = 56$
$w_2 = 22$
$w_3 = 22$

Irrigation: $w_1 = 75$

$w_2 = 25$

Rural: $w_1 = 50$

$w_2 = 25$

$w_3 = 25$

Conservation: $w_1 = 23$

$w_2 = 45$

$w_3 = 32$

The weights used for the interest groups (p) of the second data set, were kept equal for all the attributes of a particular group.

The marginal value function for a particular attribute of an interest group is assumed to be of the functional form shown below. We will for the remainder of this section, be referring to the first data set, using the forestry interest group as an example again.

$$v_{F_j}(z_{F_j}; R_{F_j}) = a - b\, e^{-z_{F_j}/R_{F_j}}$$

The parameters *a* and *b* are arbitrary in theory, but are conventionally chosen to ensure that $v_{Fj}(z_{Fj}^{min}; R_{Fj}) = 0$ and $v_{Fj}(z_{Fj}^{max}; R_{Fj}) = 1$, where z_{Fj}^{min} and z_{Fj}^{max} are respectively the worst and best values for z_{Fj}, i.e. the j–th attribute of the forestry interest group. So for the percentage change in afforestation attribute (i.e. j = 1, or column 2 of table 3.1 in section 3.2) of the forestry interest group, we will have $z_{F1}^{min} = -3$ and $z_{F1}^{max} = +5$. Furthermore, we define z_{Fj}^{mid} as the average of z_{Fj}^{min} and z_{Fj}^{max}, and in this case it turns out to be +1 (i.e. ((–3) + (+5)) / 2).

The parameter R_{Fj} is sometimes called the *"risk tolerance"*. If $v_{Fj}(z_{Fj}; R_{Fj})$ is also a utility function for decision making under uncertainty, then R_{Fj} represents the amount that a DM would be willing to give up (from the expected consequence), in order to avoid the risks associated with a particular lottery. Stewart *et al.* (1993), refer to it as being "related to the amounts that a *"rational gambler"* would be prepared to wager, if his values were in fact represented by such a function". The calculation of R_{Fj} depends on where it lies within the interval $(-\infty; +\infty)$. In the study we used an iterative procedure to calculate R_{Fj}, and this procedure is documented in appendix 3A at the end of this chapter.

We simulated different utility functions for each attribute of a particular interest group. Each function will be characterized by a single parameter u, where $u = v_{Fj}(z_{Fj}^{mid}; R_{Fj})$. Values for u are generated uniformly on [L, U]. The bounds L and U are chosen to represent specific levels of risk aversion (noting that u < ½ implies risk seeking and u > ½ implies risk aversion). For this study we used L = 0.3 and U = 0.9 as our interval bounds.

Once u has been generated, the parameters of the marginal value function $v_{Fj}(z_{Fj}; R_{Fj})$, are obtained as solutions to:

$$v_{Fj}(z_{Fj}^{min}; R_{Fj}) = 0 \tag{1}$$

$$v_{Fj}(z_{Fj}^{max}; R_{Fj}) = 1 \tag{2}$$

$$v_{Fj}(z_{Fj}^{mid}; R_{Fj}) = u \tag{3}$$

By using equations (1) and (2) to fix parameters a and b, R_{Fj} needs to be chosen to satisfy:

$$\begin{aligned} u &= v_{F_j}(z_{F_j}^{mid}; R_{F_j}) \\ &= \frac{(e^{-z_{F_j}^{min}/R_{F_j}} - e^{-z_{F_j}^{mid}/R_{F_j}})}{(e^{-z_{F_j}^{min}/R_{F_j}} - e^{-z_{F_j}^{max}/R_{F_j}})} \end{aligned}$$

This needs to be solved numerically for R_{Fj}, and the procedure used to perform this task is documented in appendix 3A at the end of this chapter. It must be remembered that for the forestry interest group, we had three separate attributes that described this interest group, and we would therefore have to find R_{Fj} for each of these relevant attributes (i.e. j = 1, 2 and 3). Therefore, we will obtain three separate marginal value functions for this interest group. Each marginal value function obtained will be multiplied by its associated importance weight w_j, and by adding these three resulting products, we can obtain the final utility function, $V_F(\mathbf{z}_F)$, for the forestry interest group.

Once we were able simulate the establishment of marginal value functions for each of the criteria of an interest group, we could calculate the values yielded by these functions for all the policy scenarios in our Background Set. We then repeated the entire process for the remaining interest groups defined for the particular data set. This would yield a set of *utility scores*, u_{ij}, for the i = 1,

..., s policy scenarios in the Background Set, and j = 1, ..., p interest groups defined for the particular data set.

The utility scores are then used to obtain the *Nash solution* (Nash: 1950), for the Background Set of policy scenarios. The Nash solution is that policy scenario that provides the "*best*" consensus solution between the p different interest groups. It will be used as a benchmark for comparing the solution(s) yielded by the particular MCDM methods, when these methods are applied to the data set at a later stage in the simulation algorithm. The formula used for obtaining the Nash value for a particular policy scenario i, is given below:

$$\prod_{j=1}^{p} \left(u_{ij} - \min(u_{ij}) \right)$$

The Nash optimum is the *maximum* solution (policy scenario), calculated over all scenarios (s) in the Background Set, of the above set of Nash values generated.

Once the utility values have been obtained, they are rescaled to lie within the range [0; 100] and ranked from largest (1) to smallest (s). This is completed separately for each interest group, and of course the Nash value with rank (1) will be the maximum and therefore the Nash solution.

3.5 The Foreground Set

The study proceeds to simulate the selection of a set of scenarios from the Background Set to form a reduced set, known as the Foreground Set of policy scenarios. Ideally, these scenarios should be chosen to contain realistically close-to-best expectations for all the interest groups, as well as some potentially good compromise solutions. We first define the attributes related to the p interest groups as the *surrogate planning objectives* z_1, ..., z_q. Recall that q = 11 for the first data set. Therefore, for this data set the following relationships will apply:

z_1 - forestry attribute 1
z_2 - forestry attribute 2
z_3 - forestry attribute 3
...
z_{11} - conservation attribute 3

For the purposes of discussion it is assumed that each z_i is *standardized* to a *maximum* value of 1 and a *minimum* value of 0. The automated approach then used in the study, is to select the Foreground Set as the t scenarios in the Background Set, that are most frequently generated according to the following algorithm:

(1) Generate random sets of weights $w_1, w_2, ..., w_q$ ($q = 11$ in our case) which sum to 1, and for each set generated find the alternative (scenario) that *minimizes* the following scalarizing function:

$$\underset{i=1}{\overset{q}{MAX}}\ w_i(z_i^* - z_i) + \varepsilon \sum_{i=1}^{q} w_i(z_i^* - z_i)$$

where z_i^* is some *"reference level"* for objective i. In our case an appropriate reference level may be the *"ideal point"* defined by $z_i^* = 1$ for all i surrogate planning objectives. We also used a small value of 0.1 (as suggested by Stewart *et al.* (1993)) for ε in the simulation study.

(2) Maintain a list of all alternatives that are generated in this way.

The number of times, out of say, 1000 randomly generated sets of weights, that each scenario was selected as *"best"* according to the above algorithm, termed the Steuer/Wierzbicki approach, was recorded. The t scenarios with the highest frequencies of being recorded as best were chosen to form the Foreground Set of policy scenarios.

We term the algorithm the Steuer/Wierzbicki approach for the following reasons. The process of reducing the Background Set to a smaller Foreground Set is analogous to the process of *filtering* defined by Steuer (1986). The difference is that Steuer developed his procedure based on alternatives that were of a linear programming structure. We have adapted his procedure and instead used the Wierzbicki (1980) scalarizing function criterion in order to select the best scenario for each set of weights generated in the algorithm. This approach was suggested in Stewart *et al.* (1993), as part of the initial iteration in their scenario based planning procedure.

The t scenarios chosen according to our algorithm, will be the ones *"most likely"* to be viewed as well balanced between the generally conflicting objectives of the various interest groups. The work of Miller (1956), suggests that t = 7 (plus or minus 2) would be an ideal number of scenarios on which value judgments, from interested parties, can effectively be expressed in terms of comparisons between them. For the purposes of discussion the number 7 will be used, although the study was conducted using Foreground Sets of sizes 5 through 9 inclusive.

3.6 Interest Thermometer Scales

Once the (t) policy scenarios that make up the Foreground Set have been calculated, we proceed with the algorithm by simulating the construction of *thermometer scales* for each interest group. These thermometer scales rank the Foreground Set scenarios according to the underlying utility preferences (scores) of the various interest groups.

In practice, each interest group would first construct thermometer scales for the criteria that are relevant to the interest. These are then aggregated to give an overall score (θ_{ij}) for the scenarios being considered. Once this has been completed, one may either display value paths (of the scenarios as scored by the various interests) for direct selection of the best two or three alternatives, say, or one could use one or more "rules" to do the selection automatically (*cf.* section 3.7 of this chapter).

We have simulated the construction of the final or aggregated (standardized) thermometer scales for the various interests (p of them), by using the (u_{ij}) utility scores associated with the Foreground Set scenarios, and rescaling them (θ_{ij}) so that the best Foreground Set alternative had a score of 100, and the worst alternative a score of 0. When doing the sensitivity analysis on our MCDM approaches, we added some "random error" to these utility scores and the results of this addition are reported on in chapters 4 and 5.

3.7 Revising the Foreground Set

We have simulated the choice of Foreground Set scenarios by using the Steuer/Wierzbicki approach (see section 3.5 of this chapter). In the simulation study, we would also like to reflect the consensus views that the DMs may have arrived at once they were able to evaluate their choices for Foreground Set scenarios. The consensus seeking process is of course an iterative one, but we will only be completing two such "*iterations*" in a single iteration of a particular simulation run for our algorithm. A single revision of the original Foreground Set (representing the second iteration in the consensus seeking process), does provide one with a good enough feel for what the process entails in reality. The manner by which we intend to simulate this, is by eliminating certain scenarios say, K, from further consideration by the DMs, and replacing them with other suitably chosen alternatives. The process of consensus seeking will be divided into two stages, namely:

(1) We will use MCDM approaches to obtain a rank ordering (across the various conflicting interests) of all the scenarios in the Foreground Set. We would then eliminate the K lowest ranking options from the set and retain the remaining ones for further consideration. The MCDM methods will be applied to the rescaled scores (θ_{ij}) of each interest group (for each Foreground Set scenario), that were previously generated in section 3.6.

(2) The second stage will essentially be a repetition of the Steuer/Wierzbicki approach (*cf.* section 3.5). The only difference is that the weights used in this step of the simulation algorithm will not lead to one or more of the eliminated scenarios being ranked more highly (according to the Wierzbicki scalarizing function criterion) than any of the scenarios classified as retained for further consideration. This is analogous to the procedures developed by Zionts and co-workers, see Zionts (1976), Zionts and Wallenius (1976, 1983) and Korhonen, Wallenius and Zionts (1984), as briefly described in section 2.2 (b) of chapter 2. This second stage of our approach was suggested in Stewart *et al.* (1993), and is represented by step 9 of the simulation algorithm provided in figure 3.1. We describe the process of determining valid sets of weights, say 1000 again, in algorithmic form below:

    (a) Let {D} = set of discarded scenarios.
    (b) Let {R} = set of retained scenarios.
    (c) Generate a set of weights.
    (d) Test to see that every scenario i in set R ranks higher, according to the scalarizing function (Wierzbicki) criterion, than every scenario j in set D.
    (e) If the test gave a true answer then count this set of weights as valid and generate a new set of weights;
    Else ignore the set of weights and generate a new set of weights.

The revised Foreground Set will therefore consist of the previously retained as well as the newly chosen policy scenarios. The latter are chosen by using the "*valid*" sets of weights.

We have labelled two steps (steps 8 and 11) of the simulation algorithm as diagrammatically depicted in figure 3.1. The first label ("*stage 1*"), corresponds to the first stage in our procedure for simulating the consensus seeking process above, whereby K scenarios are eliminated (by classification in the form of rank ordering) from the Foreground Set. We used **four** MCDM or related techniques, described in more detail below, in separate runs, in order to eliminate the K scenarios from the initial Foreground Set.

3.7.1 Maximizing the Sum of Scores (Maxscoresum)

This technique will also be referred to as the *Maxscoresum* method and it comprises two parts. The first part involves the summing of the rescaled utility scores (θ_{ij}) for each Foreground Set scenario i, where the summing takes place over the j = 1, ..., p interest groups. The second part involves the calculation of the maximum of these summed scores over all scenarios in the Foreground Set. We can combine these two parts as follows:

$$\underset{(i=1,\ldots,t)}{Max} \left[\sum_{j=1}^{p} (\theta_{ij}) \right]$$

Once the scores for each scenario have been summed, as described by the first part of this method, they are ranked from biggest to smallest. The K lowest ranking scenarios are eliminated from the Foreground Set, whilst the remaining ones are retained for further consideration.

3.7.2 Minimizing the Sum of Ranks (Minranksum)

This technique, also referred to as the *Minranksum* method, consists of two parts. The first can be described as the summing, for each Foreground Set scenario i, of the ranks of the rescaled utility scores (θ_{ij}), taken over all p interest groups. These ranks (R_{ij}) are the within-interest-group rank of the particular scenario's (i) score, as reflected by the ordinal position of this scenario on the axis of the relevant interest thermometer scale (j). Therefore, the scenario with a rescaled utility score of 100 will be awarded rank 1 for the particular interest group, and the scenario with a score of 0 will be awarded rank t. The second part consists of the calculation of the minimum of these summed ranks (since rank 1 is the best), calculated over all scenarios in the Foreground Set. Combined these two parts are shown below:

$$\underset{(i=1,\ldots,t)}{Min} \left[\sum_{j=1}^{p} R_{ij} \right]$$

Once the summed ranks (i.e. part one of the method) have been calculated, they are sorted from smallest to biggest. The scenarios that are associated with the K largest summed ranks are eliminated from the Foreground Set, whilst the remaining ones are retained.

3.7.3 **Maximum/Minimum** (Max/Min)

The *Maximum/Minimum* (or Max/Min) technique can be described as the method providing the "*best of the worst*" solution. The "*worst*" part can be described as the minimum rescaled utility score taken across all p interest groups for a particular Foreground Set policy scenario i. The "*best*" part can be described as the maximum score (of the previously worst selected ones), taken over the entire set of policy scenarios in the Foreground Set. We can combine these two parts and represent them as shown below:

$$\underset{(i=1,\ldots,t)}{Max} \left[\underset{(j=1,\ldots,p)}{Min} \left(\theta_{ij} \right) \right]$$

Once the minimum or worst scores are found for each policy scenario i (taken across the p interest groups), the K worst ones are eliminated by discarding them from the Foreground Set. The remaining scenarios are retained for further consideration.

3.7.4 ELECTRE I

The *ELECTRE I* method was described in detail in section 2.3.2 (b) of chapter 2. For our purposes, we will use ELECTRE I to find between-interest compromises. We will therefore be basing our ELECTRE I calculations on the rescaled utility values or thermometer scores, θ_{ij}, that resulted when the various interest thermometer scales were generated.

We calculated both sets of concordance and discordance values for the t Foreground Set policy scenarios. These two sets of values were used to obtain an outranking set of values (matrix) for the policy scenarios. For this study, we used $c^* = 0.5$ and $d^* = 0.5$ (in all simulation runs), as our threshold levels for concordance and discordance respectively. The outranking values would then form our outranking matrix of dimension t rows by t columns. We used this outranking matrix to obtain a partial rank ordering of the Foreground Set alternatives. The method we used to complete this rank ordering process was described in section 2.3.2 (b), and as was previously stated in this section, the method works well if there are neither too many nor too few outranking relationships. This made our choices for threshold levels critical and the values eventually used (0.5 for both concordance and discordance), were determined by experimentation.

Policy scenarios that had partial rankings of 1, 2 or 3 assigned to them were considered for further investigation and were therefore retained in the Foreground Set. We would discard the remaining ones from the Foreground Set. Clearly, this method does not produce a constant value for K, i.e. the number of alternatives to be eliminated from the Foreground Set, and K would change from one iteration to the next.

3.8 Finding the Best Policy Scenario(s)

Once the revised Foreground Set has been generated, we will simulate the procedure of setting up revised interest thermometer scales for the p interest groups. This procedure is similar to the one of setting up the initial thermometer scales, as described in section 3.6 above, and the only difference would be that a revised set of t Foreground Set policy scenarios would now be used. We will then be able to simulate the extraction of a solution set of policy scenarios from this revised Foreground Set. As discussed in section 3.7 before, the consensus seeking process consists of many iterations in reality. We have, however, only simulated 2 such iterations of this process by means of revising the initial Foreground Set at least once. We will then simulate the generation of revised interest thermometer scales, and once these have been calculated, we will simulate how DMs may choose the best scenario(s).

For this procedure we will be using the 4 MCDM methods described in sections 3.7.1 through 3.7.4 above. These methods were previously used to eliminate K scenarios from the initial Foreground Set, by using the rank orderings (of the scenarios) produced by the particular method. The only difference would be that we will on this occasion be using the particular method to simulate the search for the scenario(s) which is/are ranked first. For certain methods, this will produce a single policy scenario, whilst for others it will produce more than one scenario. Once the solution set has been selected, we can conduct a statistical analysis, in order to measure the quality of the solution produced by the particular MCDM approach.

3.9 Statistical Analysis

The statistical analysis will focus on the results produced by the 4 MCDM methods that were used at the stages labelled 1 and 2 of the simulation algorithm (see figure 3.1). Once the solution set has been generated, one would be interested to see how "*good*" a compromise solution it really provides. The term "*good*" is qualified in the analysis by measuring the quality of the solution produced against the Nash optimum. The Nash optimum is considered to be the "*pseudo-best*" or "*benchmark*" against which the other solutions could be measured. Each Foreground Set scenario has an associated ranking in terms of its Nash value, where the Nash values were generated once the Background Set of policy scenarios had been established. Clearly the scenario with ranking equal to one would be the Nash optimum. Once a solution set is chosen from the (revised) Foreground Set, one would therefore be interested to see how these solution set scenarios are ranked in relation to the Nash optimum.

Statistics were recorded for the Nash optimum solution, for each iteration of the algorithm, wherein a particular MCDM approach has been used. These are as follows:

(1) The utility value of each of the p interest groups, for the Nash optimum solution, and

(2) The standard deviation of this utility value for each of the p interest groups, for the Nash optimum solution.

Similarly, statistics were kept for the solution set produced by a particular MCDM approach, for each iteration. These are as follows:

(1) The utility value of each of the p interest groups, for the best compromise solution, and

(2) The standard deviation of this utility value for each of the p interest groups, for the best compromise solution.

Further key statistics were recorded in order to determine the quality of the best compromise solution produced. They are the following:

(1) The probability of selecting the Nash optimum as the estimated best solution (i.e. telling one how often the Nash optimum is an element of the final solution set),

(2) The average ranking of the estimated best solution (or elements of this set), where this solution is ranked according to the Nash value associated with it, and

(3) The average size of the estimated best solution set of policy scenarios.

Once we completed 100 iterations for a particular simulation run, the set of statistics we have been keeping for each iteration could be averaged for the particular population to which it applies. A final statistical analysis is therefore conducted at the end of the last iteration. We based our final judgements of the MCDM approach used in this simulation run on the statistics produced by this analysis.

APPENDIX 3A: ALGORITHMS THAT IMPLEMENT THE ITERATIVE PROCEDURE USED TO CALCULATE RISK TOLERANCE FACTORS (R_j)

The utility values for a particular interest group are generated using the following formula:

$$V(z) = \sum_{j \text{ attributes of interest}} w_j \, v_j(z_j; R_j)$$

The marginal value function $v_j(z_j; R_j)$, is evaluated for each relevant attribute j, of each interest group. The algorithm will therefore be applied to a *particular attribute* of a *specific interest group*. We will be solving for R_j (the *risk tolerance parameter*) in order to use it to obtain the marginal value function for this (j-th) attribute. The equations that are used are described in section 3.4 of chapter 3, and we are primarily interested in the following relationship:

$$v_j(z_j; R_j) = \frac{(e^{-z_j^{\min}/R_j} - e^{-z_j/R_j})}{(e^{-z_j^{\min}/R_j} - e^{-z_j^{\max}/R_j})}$$

The algorithm as described below, is divided into three sections, and section (2) will deal with the manner in which we solve for R_j.

(1) Generate a random $u \in [0; 1]$ from the Uniform distribution and transform it to lie within the pre-determined range [L; U].

(2) Check to see where u lies within the following 3 ranges and continue with the appropriate calculations.

 (a) $u \in [0.5 - \varepsilon; 0.5 + \varepsilon]$, where ε is chosen suitably (for instance a value of 0.0001 is used in the study), $\Rightarrow R_j = \infty$.

We calculate the marginal value function as follows:

$$v_j(z_j; R_j) = \frac{(z_j - z_j^{min})}{(z_j^{max} - z_j^{min})}$$

thus allowing us to find $v_j(z_j; R_j)$ for this attribute.

(b) $u > (0.5 + \varepsilon), \Rightarrow R_j > 0$.

We calculate R_j by using the following algorithm, referred to as *calc_r_rpos*():

Start with $R_j^0 = z_j^{mid} - z_j^{min}$.

(i) If $v_j(z_j^{mid}; R_j^0) < u$ (test 1)

Then

$R_j' = R_j^0$

$R_j^0 = ½ * R_j'$.

If $v_j(z_j^{mid}; R_j^0) > u$ (test 2)

Then

Proceed to *iteration*() – as described below

Else

$R_j' = R_j^0$

$R_j^0 = ½ * R_j'$

Repeat (test 2).

(ii) Else If $v_j(z_j^{mid}; R_j^0) > u$ (test 3)

Then

$R_j' = 2 * R_j^0$.

If $v_j(z_j^{mid}; R_j') < u$ (test 4)

Then

Proceed to *iteration*()

Else

$R_j^0 = R_j'$

$R_j' = 2 * R_j^0$

Repeat (test 4).

The algorithm referred to as *iteration*() is the following:

Start with the following assignments:

$\Delta_j^+ = v_j(z_j^{mid}; R_j^0) - u$

$\Delta_j^- = u - v_j(z_j^{mid}; R_j')$

$R_j^{new} = ((\Delta_j^- * R_j^0) + (\Delta_j^+ * R_j')) / (\Delta_j^- + \Delta_j^+)$

$v_j^{new} = v_j(z_j^{mid}; R_j^{new})$

(i) If $| v_j^{new} - u | < \varepsilon$

Then

Solution $R_j = R_j^{new}$.

(ii) Else If $v_j^{new} > (u + \varepsilon)$

Then

$R_j^0 = R_j^{new}$

$\Delta_j^+ = v_j(z_j^{mid}; R_j^0) - u$

$R_j^{new} = ((\Delta_j^- * R_j^0) + (\Delta_j^+ * R_j')) / (\Delta_j^- + \Delta_j^+)$

$v_j^{new} = v_j(z_j^{mid}; R_j^{new})$.

Repeat step (i).

(iii) Else If $v_j^{new} < (u - \varepsilon)$

Then

$R_j' = R_j^{new}$

$\Delta_j^- = u - v_j(z_j^{mid}; R_j')$

$R_j^{new} = ((\Delta_j^- * R_j^0) + (\Delta_j^+ * R_j')) / (\Delta_j^- + \Delta_j^+)$

$v_j^{new} = v_j(z_j^{mid}; R_j^{new})$

Repeat step (i).

(c) $u < (0.5 - \varepsilon), \Rightarrow R_j < 0$.

We calculate R_j by using the following algorithm, referred to as *calc_r_rneg()*:

Start with $R_j^0 = z_j^{min} - z_j^{max}$.

(i) If $v_j(z_j^{mid}; R_j^0) < u$ (test 1)

Then

$R_j' = R_j^0$

$R_j^0 = 2 * R_j'$.

If $v_j(z_j^{mid}; R_j^0) > u$ (test 2)

Then

Proceed to *iteration()*

Else

$R_j' = R_j^0$

$R_j^0 = 2 * R_j'$

Repeat (test 2).

(ii) Else If $v_j(z_j^{mid}; R_j^0) > u$ (test 3)

Then

$R_j' = ½ * R_j^0$.

If $v_j(z_j^{mid}; R_j') < u$ (test 4)

Then

Proceed to *iteration()*.

Else

$R_j^0 = R_j'$

$R_j' = ½ * R_j^0$

Repeat (test 4).

(3) Once we have solved for R_j in either of the algorithms described in sections b or c above, we can find the marginal value function, $v_j(z_j; R_j)$, for this attribute.

CHAPTER 4: ANALYSIS OF THE FIRST DATA SET

4.1 Procedure for Implementing the MCDM Methods

The simulation study was used to evaluate the different MCDM methods used at the steps labelled "*stage 1*" and "*stage 2*" in the simulation algorithm, as summarized in figure 3.1 of the previous chapter. This evaluation was carried out by first looking at the MCDM approach as carried out for a particular simulation run, thereby providing one with an indication of how efficient the approach is. The second part of the evaluation would focus on how robust the MCDM approach is, by carrying out a sensitivity analysis on the results yielded.

The MCDM methods were employed at the labelled stages of the simulation algorithm, for a particular simulation run, in the combinations shown below.

Stage 1 Method	**Stage 2 Method**
Maxscoresum	Maxscoresum
Minranksum	Minranksum
Max/Min	Max/Min
ELECTRE I	ELECTRE I
ELECTRE I	Maxscoresum
ELECTRE I	Minranksum
ELECTRE I	Max/Min

Once these simulation runs were completed, we could assess the results yielded by the respective combinations of MCDM methods. The efficiency of these combinations would be represented by the quality of the final solution produced by them. This means that we would be interested to see how good a compromise solution the final selection of policy scenario(s) represents for the conflicting interest groups in the study.

The sensitivity analysis will focus on **two** aspects of the simulation algorithm. The first is that of perturbing the thermometer scores of the various interest groups. The second aspect focused on was perturbing the concordance weights (of the ELECTRE I approach) assigned to the various interest groups, where initially, all interests are assigned equal weights. By perturbing the concordance weights, we were investigating the rather "*political*" issue of assigning different importance weightings to the various interest groups. We could of course also investigate this issue by perturbing the weights assigned to the interest groups for the other MCDM approaches. However, the concordance weights form an integral part of the ELECTRE I approach and it is not clear as to how random errors on these weights would affect the results produced. This makes it an important and interesting aspect of the method that needs to be addressed. For the Maxscoresum, Minranksum or Max/Min approaches, the function of the weights, and subsequent changes that could occur when specifying these weights, is much more clearly delineated. It must, however, be stated that in retrospect, one would possibly also want to investigate any random errors that could be made on these weights and this does perhaps limit the overall results yielded in this study.

On the whole, we therefore wanted to test the effects of such weight assignments on the decision making process, where errors made on these weights may conceivably have a vague or unclear effect on the result produced by the MCDM approach. This will allow us to make recommendations when other MCDM approaches are used in further studies, and where different interest weightings becomes an important and perhaps even necessary factor. We also investigated any additive or interactive effects that may result when the MCDM approaches (ELECTRE I specifically) were exposed to both the above sources of error (i.e. perturbing thermometer scores and concordance weights) simultaneously.

For the interest thermometer scores, we were interested to see just how accurately these scores had to be stipulated (i.e. how close they needed to be to their true position on the various interest groups' thermometer scales) in order to obtain the same or largely similar results when no *precision error* was involved. This approach would determine how robust the particular combination of MCDM methods would be to the degree of accuracy with which the interest groups set up their respective thermometer scales. To test the effects of such precision errors on the results, we combined the MCDM methods in the manner described below.

Stage 1 Method	**Stage 2 Method**
Maxscoresum(σ)	Maxscoresum(σ)
Minranksum(σ)	Minranksum(σ)
Max/Min(σ)	Max/Min(σ)
ELECTRE I(σ)	ELECTRE I(σ)
ELECTRE I(σ)	Maxscoresum(σ)
ELECTRE I(σ)	Minranksum(σ)

ELECTRE I(σ)	Max/Min(σ)

We used the symbol σ (sigma), for standard deviation as shown in brackets next to each method, to indicate that this MCDM method forms part of the sensitivity analysis and thereby distinguish it from its original implemented form. The true (simulated) rescaled scores, θ_{ij}, for the i-th Foreground Set policy scenario (i = 1, ..., t) of the j-th interest thermometer scale (j = 1, ..., p), were perturbed by the *addition* of a *precision error* in the form of the random variable, z_{ij}. The z_{ij} are independently identically distributed (i.i.d.) random variables from the Normal distribution with a mean value of 0 and a variance of σ^2. The resulting perturbed scores for a particular thermometer scale were again rescaled, so that the maximum had a value of 100 and the minimum a value of 0. We will provide the exact figures used for σ when tabulating the results of the simulation runs for the sensitivity analysis in appendix 4B at the end of this chapter.

The second area of the sensitivity analysis would look at the issue of perturbing the concordance weights that were used in the ELECTRE I method. Our starting point was that each interest group had an equal weight and we could then proceed to perturb these weights *multiplicatively*. This was completed by combining the MCDM methods in the manner shown below.

Stage 1 Method	**Stage 2 Method**
ELECTRE I(α)	ELECTRE I(α)
ELECTRE I(α)	Maxscoresum
ELECTRE I(α)	Minranksum
ELECTRE I(α)	Max/Min

The symbol α (alpha), that appears in brackets next to the ELECTRE I methods, is used to indicate that they form part of the sensitivity analysis. The weight, w_j, for the j-th interest group is perturbed multiplicatively as follows:

$$\hat{w}_j = w_j * e^{u_j}$$

where the u_j are i.i.d. random variables from the Normal distribution, with mean 0 and variance σ^2.

We obtain this functional form from the equation shown below, where u_j is the *proportional error* to which the weight w_j is subjected:

$$\log_e \hat{w}_j = \log_e w_j + u_j$$

In this study we suppose that the magnitude of the proportional error is at worst 100α%. Restating this in more precise terms would be that an approximate 95% confidence interval, is ± 100α% when using a 2 standard deviation approximation. The following set of equations will demonstrate what this implies and how α is used in our calculations:

$$\Rightarrow \quad \frac{1}{1+\alpha} < e^{u_j} < 1+\alpha$$

$$where \quad e^{u_j} \in [e^{-2\sigma}; e^{+2\sigma}]$$

$$So \quad 1+\alpha \approx e^{+2\sigma}$$

$$\therefore \quad \sigma \approx \tfrac{1}{2} \log_e (1+\alpha)$$

From the above set of equations we can see how α is used to provide us with the standard deviation σ, which in turn is used to calculate the proportional error, u_j. We will provide the exact figures used for α when tabulating the results of the simulation runs for the sensitivity analysis in appendix 4B at the end of this chapter.

We also combined the two potential sources of error, by perturbing both the concordance weight assignments as well as the thermometer scores. The manner in which the MCDM methods were combined in order to complete this part of the sensitivity analysis is shown below.

Stage 1 Method	**Stage 2 Method**
ELECTRE I(α)(σ)	ELECTRE I(α)(σ)
ELECTRE I(α)(σ)	Maxscoresum(σ)
ELECTRE I(α)(σ)	Minranksum(σ)

ELECTRE I(α)(σ) Max/Min(σ)

We will again provide the exact figures used for α and σ in appendix 4B at the end of this chapter.

As mentioned before, the sensitivity analysis would test how robust the respective combination of MCDM methods were, by focusing primarily on two potential sources of error, *viz.*, the interest thermometer scores and the concordance weights assigned to the interest groups. These sensitivity tests were, however, not carried out on all the Foreground Set sizes used in the initial stage of the study (i.e. sizes 5 through 9), wherein we determined the efficiency of the various MCDM approaches. A Foreground Set consisting of t = 7 scenarios would suffice for the test purposes since this was also the number suggested according to the work of Miller (1956) – see the end of section 2.2 of chapter 2 and section 3.5 of chapter 3. We will therefore base our conclusions on the results yielded by the sensitivity runs for 7 Foreground Set scenarios. However, we will also consider what the average results were for all the Foreground Set sizes, when we previously assessed the efficiency of the various MCDM approaches.

4.2 Results Produced by the MCDM Methods

The results that provide the detailed outcome of the simulation runs conducted for the various MCDM approaches, are tabled in appendix 4A at the end of this chapter. The description of these results for the particular combination of MCDM methods is given below, with each description containing a reference (in its heading) to the table number in appendix 4A to which it refers.

Before dealing with the individual combinations of methods, it is worth noting that the forestry interest group appears to be disadvantaged when looking at the mean utility values of the Nash solution, for all 7 combinations of MCDM methods used. These mean utility values for the 7 combinations of methods are provided at the top of tables 4A.1 to 4A.7 in appendix 4A. An example of this discrepancy may be found in table 4A.1, where forestry has an average utility value for the Nash Solution of 60.83, versus values of 67.30, 67.52 and 67.16 for the remaining interests. The reason for this seeming anomaly, is that forestry is more fundamentally in opposition to the other three interests in this data set. This would therefore imply that it becomes increasingly more difficult to obtain a good compromise solution.

For this data set, we did not vary the degree of overlap for the various attributes that applied to the four interest groups. This may have some bearing on the anomaly that has occurred and is perhaps a restricting limitation for the analysis on this data set. Therefore, when doing the simulation process for the second (randomly generated) data set, we will vary the attributes, together with their directions of preference. Consequently, the analysis on the results obtained from this data set in chapter 5 (i.e. the mean utilities for the Nash optimum at least) should not favour any particular interest group, and a fair spread of the mean utility values across all interests is expected to occur.

Maxscoresum + Maxscoresum (table 4A.1)

We see that this combination of MCDM methods provides us with a probability of 72% (on average) of selecting the Nash optimum as our solution. Coupled to this, the solution is a single best policy scenario which has an average ranking of 1.86. Furthermore, the utility values of the respective interest groups in our solution compare favourably to those of the Nash optimum (provided at the top of the table), which was used as a basis for comparison. We also see that these utility values represent a fair compromise solution to all groups, since they on average range in utility preference from approximately 62 (62.22) to 69 (68.94). These averages for the compromise solution lie within 1 to 3 units (on the 0–100 scale) of the Nash optimum (i.e. within 2 to 5% of Nash).

When considering the probability of selecting the Nash optimum as our solution, one can see that it tends to increase as the Foreground Set increases in size. There is, however, no noticeable effect for this statistic when changing the value for K, i.e. the number of scenarios discarded whilst revising the Foreground Set.

Minranksum + Minranksum (table 4A.2)

The probability of selecting the Nash optimum as our solution (or as is the case for this combination, an element of the solution set) is on average only 56%, which is substantially worse than, say, the 72% reported for the Maxscoresum + Maxscoresum approach above. Furthermore, in a solution set consisting on average of 1.33 alternatives, we find that the scenarios have an average ranking equal to 3.13. The utility values for the solution produced by this combination of MCDM methods, on average tend to favour the rural communities interest group (average utility = 68.91). We also find that there exists a more noticeable difference between the average utility values of the compromise solution versus those of the Nash solution for the remaining interest groups. These average values lie within 2 to 6 units (on the 0–100 scale) or 3 to 9% of the Nash optimum.

There are no noticeable effects in the results when changing either the size of the Foreground Set or the number of scenarios discarded (K) when revising the Foreground Set.

Max/Min + Max/Min (table 4A.3)

This combination produces a single best solution and we have on average a 61% probability of selecting the Nash optimum as this solution. The policy scenario in the solution set has an average ranking equal to 1.72. The average utility values for the forestry (58.51) and irrigation (62.45) interest groups is noticeably lower than those of the remaining two interest groups. These same two interests also have lower average utilities for the compromise solution when compared to the Nash optimum. The average utility values for the compromise solution lie within 2 to 6 units (on the 0–100 scale) or 3 to 9% of the Nash optimum.

There are no noticeable effects in the results when changing either the size of the Foreground Set or the number of policy scenarios discarded (K) when revising the Foreground Set.

ELECTRE I + ELECTRE I (table 4A.4)

This combination of MCDM methods produces on average a 72% probability of selecting the Nash optimum as part of the solution set. This is one of the highest probabilities reported for the combinations of methods and is matched only by the Maxscoresum + Maxscoresum approach. However, to place this in perspective, we see that the solution consists, on average, of 2.01 policy scenarios. Coupled to this, the scenarios in the solution set have an average ranking equal to 3.70. Furthermore, the rural communities interest group is favoured by this combination of methods with an average solution utility value of 69.18, when compared to the remaining interest groups (utility values of 58.53, 59.01 and 63.31). The remaining interests also have noticeably lower average utilities for the compromise solution when compared to the Nash optimum. These average utilities lie within 2 to 8 units (on the 0–100 scale) of the Nash optimum, or put differently, within 3 to 12% of Nash.

When considering the probability of selecting the Nash optimum as part of the solution set, one can see that, with the exception of a Foreground Set of size 7 scenarios, this probability increases as the value for C increases. In other words, by retaining more scenarios when revising the Foreground Set, we increase the probability of selecting the Nash solution (as part of our solution set). There are, however, no marked effects on the results when looking at the different sizes of the Foreground Sets.

ELECTRE I + Maxscoresum (table 4A.5)

The probability of selecting the Nash optimum as our solution, where this solution consists of only a single policy scenario, is on average 71%. This is a high probability when considering that there is only a single alternative in the solution. Furthermore, this scenario has an average ranking of 1.79. The utility values for the alternatives in our solution set are close to those of the Nash solution. These average utility preferences also range from approximately 60 (60.44) to 70 (69.61) for the different interest groups and lie within 1 to 3 units (on the 0–100 scale) or 1 to 4% of the Nash optimum.

There are no marked effects in the results when changing the number of scenarios retained for further consideration (C), when we revise the Foreground Set. Different Foreground Set sizes on the whole also tend to play no major role in determining the best solution. We do, however, find that when there is a Foreground Set of size 7 policy alternatives, both the probability of selecting the Nash optimum and, more so, the average ranking of the solution tends to worsen. This deterioration can be seen when we compare these two statistics for the other Foreground Set sizes used in the study.

ELECTRE I + Minranksum (table 4A.6)

This combination of MCDM approaches produces a solution that has an average size of 1.25 policy alternatives. There is on average a 57% chance of selecting the Nash optimum as part of the solution set and the scenarios in the set will have an average ranking equal to 2.60. The rural communities interest group is favoured by the solution produced by this combination of methods, with an average solution utility value of 71.02. This one can compare to the remaining interest groups that have average utility values of 60.53, 61.54 and 62.71. The average utility values for the compromise solution tend to lie within 1 to 5 units (on the 0–100 scale) or 1 to 8% of the Nash optimum.

It is noted that the Foreground Set of size 6 and a partial rank cut off value (C) of 3, produces a solution that has a probability of including the Nash optimum in the solution set equal to 73%. This is a markedly higher probability than all the other Foreground Set sizes or partial rank cut off values. There are no other noticeable effects reflected in the results when either changing the Foreground Set size or the partial rank cut off values (C).

ELECTRE I + Max/Min (table 4A.7)

For this combination of methods, the probability of selecting the Nash optimum as the solution is on average 61%, where this solution tends to be a single policy scenario. This scenario has an average ranking of 1.77. The forestry interest group appears to be disadvantaged by the solution produced for this combination of methods, with an average solution utility value of only 57.17. This compares to the remaining interest groups that have average solution utility values of 62.13, 68.14 and 66.87. These average utilities for the compromise solution lie within 1 to 5 units (on the 0–100 scale) or 2 to 8% of the Nash optimum.

There are no noticeable effects in the results when changing the partial rank cut off value (C), but we do see that the probability of selecting the Nash optimum tends to increase when larger Foreground Set sizes are used.

4.3 Sensitivity Analysis

The sensitivity analysis focused on perturbing firstly, the interest thermometer scores, followed by perturbing the concordance weights assigned to the interest groups. These were then combined so that both sources of error could be investigated simultaneously, in a particular simulation run.

We have previously stated at the end of section 4.1, that the sensitivity runs were conducted on the different combinations of MCDM methods using Foreground Sets of size 7 scenarios only. The results for these sensitivity runs are tabulated in appendix 4B at the end of this chapter, and will therefore be obtained by using Foreground Sets consisting of 7 policy scenarios only. We will again accompany the description of these results with a reference (in the heading of the description) to the table number in appendix 4B to which it refers.

4.3.1 Perturbing the Interest Thermometer Scores

Maxscoresum(σ) + Maxscoresum(σ) (table 4B.1)

For K = 3, there appears to be some deterioration of the results when compared to those corresponding to them in table 4A.1. This is reflected mainly in the deterioration of the probability of selecting the Nash optimum when σ changes in value from 5 (0.69) to 10 (0.59). For K = 4, the probability of selecting the Nash optimum worsens even more when σ changes from 5 (0.72) to 10 (0.48).

We also find that the utility values for the compromise solution tend to lie within 3 to 8 units (on the 0–100 scale) or 4 to 12% of the Nash optimum. This is more than the 2 to 5% recorded for the unperturbed results in table 4A.1, and would seem to suggest that this combination of methods is not robust when there is a large form of precision error made on the interest thermometer scores.

Minranksum(σ) + Minranksum(σ) (table 4B.2)

There are no changes of note in these results (i.e. deteriorating outcomes) when compared to the corresponding results obtained in table 4A.2. This combination of MCDM methods therefore appears to be robust to precision errors being made on the interest thermometer scores.

Max/Min(σ) + Max/Min(σ) (table 4B.3)

There is no marked deterioration in these results when the scores are perturbed (compared to the corresponding results in table 4A.3), and this combination of methods is therefore seemingly robust to errors being made on the thermometer scores.

ELECTRE I(σ) + ELECTRE I(σ) (table 4B.4)

The probability of selecting the Nash optimum and the average ranking of the scenarios in the solution set, deteriorated noticeably (for $\sigma = 10$) when compared to the unperturbed set of results in table 4A.4. This deterioration is visible for both C = 2 and 3. Original figures for these statistics were (from table 4A.4) for C = 2: 0.7 and 3.98 respectively and for C = 3: 0.71 and 3.73 respectively.

We also find that the utility values for the compromise solution tend to lie within 2 to 13 units (on the 0–100 scale) or 3 to 19% of Nash. This is more than the 3 to 12% recorded for the unperturbed results in table 4A.4, and would seem to lend weight to the suggestion that this combination of MCDM methods is not robust to precision errors being made on the interest thermometer scores.

ELECTRE I(σ) + Maxscoresum(σ) (table 4B.5)

There is a noticeable decrease in the probability of selecting the Nash optimum for $\sigma = 10$ (for C = 3), when these results are compared to the unperturbed results in table 4A.5. The utility values for the compromise solution tend to lie within 2 to 5 units (on the 0–100 scale) or 3 to 8% of the Nash optimum. This compares favourably with the 1 to 4% recorded for the unperturbed results in table 4A.5. Based on these results, we can therefore not conclude with any certainty as to whether or not this combination of MCDM methods is robust to precision errors being made on the thermometer scores.

ELECTRE I(σ) + Minranksum(σ) (table 4B.6)

The results shown in this table indicate that no marked deterioration of the compromise solution has occurred when compared to the unperturbed results of table 4A.6. This, however, only applies to the probability of selecting the Nash optimum and the average ranking of solution set alternatives.

The utility values for the compromise solution tend to lie within 2 to 10 units (on the 0–100 scale) or 3 to 15% of the Nash optimum. This is noticeably more than the 1 to 8% recorded for the unperturbed results in table 4A.6. Based on these results, we can therefore conclude that this combination of MCDM methods is not robust to precision errors being made on the thermometer scores.

ELECTRE I(σ) + Max/Min(σ) (table 4B.7)

There are noticeable changes that result when the thermometer scores are perturbed for this combination of methods. These changes are visible when one looks at the probability of selecting the Nash optimum as well as the average ranking of the solution policy scenario. Both changes that have occurred constitute a deterioration of the results and this happens when $\sigma = 10$ for $C = 2$ and $\sigma = 5$ for $C = 3$.

The utility values for the compromise solution lie within 3 to 10 units (on the 0–100 scale) of the Nash optimum, compared to 1 to 5 units for the unperturbed results. Therefore, when compared to the unperturbed results of table 4A.7, we find that this combination of MCDM methods does not seem to be robust to certain levels of error being made on the interest thermometer scores.

4.3.2 Perturbing the Concordance Weights

The second part of the sensitivity analysis looked at perturbing the concordance weights, used in the ELECTRE I method, that were assigned to each interest group.

ELECTRE I(α) + ELECTRE I(α) (table 4B.8)

For $C = 2$, there is a change in the average ranking of the solution set scenarios from the first level of α introduced. The unperturbed value for this statistic was 3.98 in table 4A.4, and we see that at level $\alpha = 0.05$ it already has a value of 4.99, later deteriorating to a value of 6.11 for α = 0.40. There is also a deterioration of the probability to selecting the Nash optimum, where the unperturbed probability was 0.7 in table 4A.4.

However, for C = 3 these effects are less noticeable. The average ranking of the solution set scenarios do, however, worsen markedly when α reaches the level of 0.15. This combination of MCDM methods therefore provides one with a signal representing a complex interaction. The utility values for the compromise solution tend to lie within 5 to 13 units (on the 0–100 scale) or 7 to 19% of the Nash optimum. This is considerably more than the 3 to 12% recorded in table 4A.4. We can therefore conclude that the evidence provided does seem to indicate that this combination of methods is not robust to proportional errors being made on the concordance weights.

ELECTRE I(α) + **Maxscoresum** (table 4B.9)

The proportional errors that were simulated for the concordance weights do not appear to affect these results negatively when they are compared to the unperturbed results provided in table 4A.5. The utility values for the compromise solution tend to lie within 3 to 6 units (on the 0–100 scale) of the Nash optimum, compared to the 1 to 3 units for the unperturbed results. Therefore, this combination of MCDM methods seems to be robust to any errors being made on the concordance weights.

ELECTRE I(α) + **Minranksum** (table 4B.10)

The utility values for the compromise solution tend to lie within 2 to 10 units (on the 0–100 scale) or 3 to 15% of the Nash optimum. This is noticeably more than the 1 to 8% recorded for the unperturbed results in table 4A.6. Therefore, this combination of MCDM methods seems not to be robust to any errors being made on the concordance weights.

ELECTRE I(α) + **Max/Min** (table 4B.11)

There appears to be no marked deterioration in these results when compared to those obtained for the unperturbed simulation runs in table 4A.7. The utility values for the compromise solution tend to lie within 3 to 7 units (on the 0–100 scale) of the Nash optimum, compared to the 1 to 5 units for the unperturbed results. Therefore, this combination of MCDM methods seems to be robust to proportional errors being made on the concordance weights.

4.3.3 Perturbing both the Interest Thermometer Scores and the Concordance Weights

The final part of the sensitivity analysis consisted of combining the precision errors on the thermometer scores with the proportional errors on the concordance weights, in a single simulation run. The key issue we wanted to address by doing this, is to find out whether these effects appear to be simply additive or whether there is in fact evidence of some form of interaction between the two sources of error.

ELECTRE I(α)(σ) + ELECTRE I(α)(σ) (table 4B.12)

This combination of MCDM methods did not appear to be robust when previously exposed to both precision and proportional errors in separate sensitivity runs. The results shown in this table indicate that there is some evidence of an additive effect when the combination of methods is exposed simultaneously to both sources of error. When we look at the average ranking of the scenarios in the solution set in particular, there is an indication that this combination of MCDM methods is not robust when exposed to both sources of error simultaneously. This is further supported by the utility values of the compromise solution. In this set of runs they tend to lie within 9 to 27% of the Nash optimum, compared to the 3 to 12% for the unperturbed simulation runs in table 4A.4.

ELECTRE I(α)(σ) + Maxscoresum(σ) (table 4B.13)

For the previous sensitivity tests, this combination of methods seemed to be robust when tested on proportional errors made on the concordance weights, but it was not clear as to whether or not this combination was robust when tested separately on the precision errors made on the thermometer scores. We now find, after combining these two sources of error in the same sensitivity runs, that for C = 2, the results worsen when looking at the probability of selecting the Nash optimum in particular. However, for C = 3, the results do not appear to be much different from those obtained in the previous two (separate) sensitivity tests. This combination of MCDM methods therefore provides one with evidence of some form of interaction between the two sources of error when they are combined in the same set of sensitivity runs.

Furthermore, the utility values for the compromise solution tend to lie within 3 to 11% of the Nash optimum, compared to the 1 to 4% for the unperturbed results in table 4A.5.

Based on these results, we therefore cannot be sure whether or not this combination is robust when exposed to both sources of error simultaneously.

ELECTRE I(α)(σ) + Minranksum(σ) (table 4B.14)

This combination of MCDM methods seemed not to be robust in both previous sets of sensitivity runs when they were conducted in separate simulation runs. After combining these two sources of error in the same sensitivity runs, we find that for C = 2, there is a deterioration of the results for both the probability of selecting the Nash optimum and the average ranking of the policy scenarios in the solution set. However, for C = 3, there appears to in fact be an improvement in these results when compared to the unperturbed results obtained from table 4A.6.

Furthermore, the utility values for the compromise solution tend to lie within 1 to 7 units (on the 0–100 scale) or 2 to 11% of the Nash optimum. This compares favourably with the 1 to 8% recorded for the unperturbed results. There appears to be some form of interaction between these two sources of error and based on this evidence, we therefore cannot be sure whether or not this combination of methods is robust when exposed to both sources of error simultaneously.

ELECTRE I(α)(σ) + Max/Min(σ) (table 4B.15)

This combination of MCDM methods seemed to be robust when subjected to proportional errors made on the concordance weights, but not robust when subjected to precision errors made on the interest thermometer scores (when previously tested in separate sensitivity runs). We now see that the results shown in this table are definitely worse than the results obtained when this combination of methods was exposed to both sources of error in separate sensitivity runs. This is the case for both C = 2 and 3 and is reflected when looking at both the probability of selecting the Nash optimum as well as the average ranking of the solution.

The effects of these two sources of error appears to be additive and this combination of MCDM methods is therefore not robust when exposed to both sources of error in the same sensitivity runs. This is further supported by the utility values of the compromise solution. In this set of runs they tend to lie within 2 to 19% of the Nash optimum, compared to the 2 to 8% for the unperturbed simulation runs in table 4A.7.

4.4 Overall Conclusions for the Analysis on this Data Set

In table 4.1 on the following page, we provide the summarized findings of the evaluation of the various combinations of MCDM methods, as previously discussed in sections 4.2 and 4.3. This table provides information for the various combinations concerning: (i) the combination's performance measure or quality of the solution produced, (ii) whether or not the combination is affected by the size of the Foreground Set and K or C, and (iii) how robust the method is when exposed to the two forms of error as implemented by the sensitivity analysis.

The overall conclusions that can be drawn from our findings are the following:

(i) From the point of view of quality of solution that the combination of methods produce, we would definitely recommend the Maxscoresum + Maxscoresum and the ELECTRE I + Maxscoresum MCDM approaches. We would possibly also recommend the Max/Min + Max/Min and ELECTRE I + Max/Min approaches. The ELECTRE I + ELECTRE I method fared rather poorly when ranking the solution set scenarios produced, and we therefore hesitate to recommend this combination of methods without further investigating why this has occurred.

(ii) From the point of view of robustness of the combination of methods, we would definitely recommend the Max/Min + Max/Min MCDM approach, and possibly also the Minranksum + Minranksum, ELECTRE I + Maxscoresum and Maxscoresum + Maxscoresum approaches. The latter combination would be recommended because it was not robust for only very large precision errors being made on the thermometer scores.

(iii) On the whole, there does not appear to be a preferred Foreground Set size or preferred values for K or C according to the results obtained.

(iv) Therefore from an overall point of view (combining mainly criteria (i) and (ii) above) we would tend to favour the Max/Min + Max/Min and Maxscoresum + Maxscoresum MCDM approaches. However, the reason why the Maxscoresum + Maxscoresum approach failed to be robust for large forms of precision error being made on the thermometer scores, needs further investigation.

TABLE 4.1: Summary of Results: Data Set No. 1

Combination of Methods	Combination Performance as Measured by:		Results for Combination Affected by:		How Robust is the Combination when Exposed to:		
	Probability of Selecting the Nash Optimum	Average Ranking of Solution Set Scenarios	Foreground Set Size	K or C	Precision Error (thermometer scores)	Proportional Error (concordance weights)	Precision and Proportional Error
Maxscoresum + Maxscoresum	72%	1.86	yes	no	not robust	n/a	n/a
Minranksum + Minranksum	56%	3.13	no	no	robust	n/a	n/a
Max/Min + Max/Min	61%	1.72	no	no	robust	n/a	n/a
ELECTRE I + ELECTRE I	72%	3.70	no	yes	not robust	not robust	not robust
ELECTRE I + Maxscoresum	71%	1.79	inconclusive	no	not clear	robust	not clear
ELECTRE I + Minranksum	57%	2.60	inconclusive	inconclusive	not robust	not robust	not clear
ELECTRE I + Max/Min	61%	1.77	yes	no	not robust	robust	not robust

It must be noted that at this stage of the study, we have only analyzed the results obtained from the first data set. Our recommendations will possibly change once we have analyzed the results obtained from the second randomly generated data set. These results are analyzed in the following chapter.

APPENDIX 4A: THE DETAILED RESULTS OF THE VARIOUS MCDM APPROACHES FOR THE FIRST DATA SET

In this appendix we provide the detailed tabulated outcomes of the various MCDM approaches as these combinations of methods were applied to the first data set. The tables listed below have column headings that are too long to be given in full. Therefore, the columns are headed by shortened descriptions. Their full descriptions are given below and they apply to all the tables in this, and subsequent appendices.

Foregr. Set: The size (number of policy scenarios, t) of the Foreground Set.

K: The number of policy scenarios that will be eliminated from the Foreground Set before revising this set.

C: It is applicable whenever the ELECTRE I method was used. Scenarios with a partial ranking less than or equal to C assigned to them would be retained for further consideration, whilst the remaining ones would be eliminated when revising the Foreground Set.

The statistics that are kept relating to the Nash optimum solution are the following:

1. Mean Utility Values for the (4) Interest Groups and their associated Standard Deviations: These utility values are the averages taken over all Foreground Set sizes and their respective values of K or C (relating to the Nash solution), for a particular combination of MCDM methods (i.e. for a particular table of results). At the top of each table we therefore have the mean utility values for each of the four interest groups (their order being: forestry, irrigation, rural communities and conservation), together with their associated standard deviations. The initial values used to obtain the mean utility values of the Nash optimum solution, are taken from the population consisting of 100 iterations, where an iteration is described by the simulation algorithm summarized in figure 3.1 of chapter 3. The initial utility value used to calculate the mean shown at the top of each table, is therefore also an average that has been calculated for the 100 iterations of a particular simulation run. In each iteration we will therefore have a unique set of utility values that have been generated for each relevant attribute of a particular interest group.

The statistics that are kept relating to the best compromise solution are the following:

2. Mean Utility Values for the (4) Interest Groups and their associated Standard Deviations: The population that was used to calculate these mean utility values for each of the four interest groups, again consists of 100 iterations (i.e. one simulation run), where a single iteration is described by the simulation algorithm summarized in figure 3.1 of chapter 3. For these statistics, the utility values are calculated for the best compromise solution in each iteration of a particular simulation run.

Probability or Prob.: The probability of selecting the Nash optimum as the best solution (i.e. telling us how often the Nash optimum is an element of the final solution set). This probability is calculated for 100 iterations of a particular simulation run.

Average Rank: The average ranking of the best solution or elements of this solution set, where the ranking takes place according to the Nash value associated with the policy scenario(s). The average is calculated for 100 iterations of a particular simulation run.

Soln. Set Size: The average size of the best solution set of policy scenarios. This average is calculated for 100 iterations of a particular simulation run.

Average: The average column values (i.e. the mean utility values for the (4) interest groups, the probability of selecting the Nash optimum, etc.) for the best compromise solution. This average is calculated for all Foreground Set sizes and their respective values of K or C. The average utility values of the four interest groups calculated for the best compromise solution here, therefore corresponds to the average utility values of the Nash optimum solution shown at the top of each table.

The tables for the various combinations of MCDM methods are given below.

TABLE 4A.1: Results of Maxscoresum + Maxscoresum Simulation Runs

					1. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Nash Solution							
					60.83	2.2	67.30	2.9	67.52	2.9	67.16	2.9
Foregr. Set	**K**	**Probability**	**Average Rank**	**Soln. Set Size**	**2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods**							
5	2	0.69	2.1	1	59.27	2.1	63.41	2.7	72.09	3.3	65.54	2.8
5	3	0.67	1.74	1	62.38	2.4	64.41	2.8	69.92	3.1	66.87	2.9
6	2	0.68	1.89	1	60.95	2.3	64.67	2.7	71.58	3.3	65.75	2.9
6	3	0.59	2.14	1	61.19	2.3	65.42	2.9	67.32	2.9	64.02	2.7
6	4	0.68	2	1	61.44	2.3	65.68	2.8	67.25	2.9	66.18	2.9
7	2	0.74	1.99	1	62.97	2.5	65.98	2.9	70.16	3.2	64.26	2.7
7	3	0.68	2.49	1	63.41	2.6	68.56	3.1	68.28	3.0	61.75	2.5
7	4	0.74	1.73	1	62.22	2.4	64.63	2.7	68.77	3.0	65.95	2.8
7	5	0.73	1.74	1	63.45	2.5	68.58	3.1	67.29	2.8	63.57	2.6
8	2	0.73	2.06	1	63.31	2.5	69.53	3.2	68.71	3.0	63.04	2.6
8	3	0.77	1.55	1	61.96	2.4	67.63	3.0	67.95	2.9	65.03	2.7
8	4	0.68	2.15	1	63.25	2.5	65.33	2.8	69.36	3.1	65.03	2.8
8	5	0.7	2.04	1	63.45	2.6	67.4	3.0	69.02	3.0	63.22	2.6
8	6	0.71	1.91	1	64.29	2.6	64.13	2.8	71.08	3.2	62.48	2.6
9	2	0.82	1.52	1	62.87	2.5	69.24	3.1	68.56	3.0	63.38	2.6
9	3	0.77	1.4	1	60.47	2.2	67.26	3.0	69.61	3.1	68.17	3.0
9	4	0.79	1.67	1	59.18	2.1	67.33	3.0	67.04	2.8	69.01	3.1
9	5	0.76	1.79	1	63.26	2.5	68.98	3.1	67.06	2.9	64.43	2.7
9	6	0.75	1.63	1	62.4	2.4	66.91	2.9	69.59	3.1	64.72	2.8
9	7	0.74	1.72	1	62.64	2.5	68.11	3.0	68.25	3.0	64.07	2.7
Average		0.72	1.86	1.00	62.22	2.4	66.66	2.9	68.94	3.0	64.82	2.8

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4A.2: Results of Minranksum + Minranksum Simulation Runs

					1. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Nash Solution							
					60.97	2.2	67.03	2.9	67.42	2.9	66.44	2.8
Foregr. Set	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
5	2	0.62	2.71	1.43	59.66	2.1	62.59	2.6	69.82	3.1	64.58	2.7
5	3	0.67	3.01	1.61	60.5	2.2	57.64	2.1	72.75	3.4	62.03	2.4
6	2	0.61	3.17	1.45	62.71	2.5	63.59	2.6	70.72	3.2	58.38	2.2
6	3	0.59	2.82	1.34	62.49	2.4	58	2.2	72.19	3.3	62.12	2.5
6	4	0.55	2.63	1.19	58.52	2.1	62.96	2.6	69.05	3.0	65.79	2.9
7	2	0.57	2.92	1.41	63.95	2.6	58.81	2.3	71.15	3.3	60.06	2.2
7	3	0.58	2.97	1.41	63.17	2.5	64.28	2.7	68.17	3.0	59.22	2.2
7	4	0.54	3.15	1.28	61.25	2.4	64.81	2.8	69.53	3.1	61.67	2.5
7	5	0.43	3.93	1.34	65.56	2.7	61.63	2.6	66.51	2.9	56.25	1.9
8	2	0.6	2.69	1.27	64.02	2.6	60.39	2.4	71.88	3.4	61.63	2.5
8	3	0.59	2.53	1.22	61.14	2.4	63.98	2.7	70.53	3.2	63.44	2.6
8	4	0.57	3.58	1.31	64.67	2.6	67.5	3.0	66	2.8	59.97	2.4
8	5	0.5	3.50	1.3	65.3	2.7	63.82	2.8	66.19	2.9	58.03	2.1
8	6	0.58	3.28	1.38	61.22	2.3	62.14	2.6	68.41	3.0	61.4	2.5
9	2	0.6	2.79	1.24	64.32	2.6	67.04	3.0	67.75	2.9	59.85	2.3
9	3	0.6	3.31	1.31	64.1	2.6	63.96	2.7	67.79	3.0	59.16	2.2
9	4	0.52	3.54	1.3	64.47	2.6	65.68	2.9	67.29	2.9	58.02	2.2
9	5	0.43	3.27	1.28	65.43	2.7	60.88	2.6	66.86	2.9	59.53	2.3
9	6	0.47	3.73	1.28	65.08	2.7	62.64	2.7	68.3	3.0	56.99	2.1
9	7	0.54	3.01	1.26	63.16	2.5	61.9	2.6	67.23	2.9	63.25	2.7
Average		0.56	3.13	1.33	63.04	2.5	62.71	2.6	68.91	3.1	60.57	2.4

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4A.3: Results of Max/Min + Max/Min Simulation Runs

Foregr. Set	K	Probability	Average Rank	Soln. Set Size	1. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Nash Solution							
					61.12	2.3	67.14	2.9	67.94	2.9	66.45	2.8
					2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
5	2	0.5	1.84	1	55.33	1.7	61.23	2.3	68.81	3.0	65.61	2.7
5	3	0.51	1.93	1	57.27	1.9	59.16	2.2	68.63	3.0	67.57	2.9
6	2	0.59	1.7	1	59.12	2.1	65.3	2.7	67.25	2.8	64.41	2.6
6	3	0.7	1.59	1	60.96	2.2	63.86	2.6	67.98	2.9	64.36	2.6
6	4	0.57	1.95	1	59.84	2.1	57.81	2.1	69.03	3.0	67.83	2.9
7	2	0.68	1.61	1	60.47	2.1	63.8	2.6	67.16	2.8	65.6	2.7
7	3	0.59	1.75	1	56.79	1.8	64.76	2.7	66.47	2.8	66.3	2.8
7	4	0.52	1.82	1	59.83	2.1	61.79	2.4	64.24	2.5	65.56	2.7
7	5	0.63	1.64	1	61.28	2.3	60.31	2.3	67.86	2.9	66.89	2.8
8	2	0.65	1.64	1	55.41	1.7	63.05	2.5	65.04	2.6	71.83	3.3
8	3	0.67	1.71	1	56.79	1.8	62.84	2.5	65.23	2.7	69.11	3.0
8	4	0.67	1.53	1	62.45	2.4	61.4	2.4	67.07	2.8	65.75	2.7
8	5	0.57	1.73	1	60.95	2.2	60.88	2.3	64.77	2.6	63.5	2.5
8	6	0.62	1.73	1	59.17	2.0	63.45	2.5	65.95	2.7	66.75	2.8
9	2	0.61	1.76	1	55.4	1.7	64.46	2.6	65.06	2.6	70.82	3.2
9	3	0.56	1.83	1	55.88	1.7	63.35	2.5	64.08	2.5	69.42	3.1
9	4	0.62	1.6	1	58.78	2.0	64.94	2.7	63.99	2.5	67.25	2.9
9	5	0.61	1.67	1	61.02	2.2	61.32	2.4	66.66	2.8	65.26	2.7
9	6	0.61	1.83	1	57.43	1.9	61.7	2.4	64.7	2.6	69.01	3.1
9	7	0.62	1.56	1	55.98	1.7	63.56	2.6	67.41	2.8	70.47	3.2
Average		0.61	1.72	1.00	58.51	2.0	62.45	2.5	66.37	2.7	67.17	2.9

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4A.4: Results of ELECTRE I + ELECTRE I Simulation Runs

Foregr. Set	C	Probability	Average Rank	Soln. Set Size	1. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Nash Solution							
					60.61	2.2	66.92	2.9	67.30	2.8	66.36	2.8
					2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
5	2	0.72	3.86	2.1	54.52	1.6	52.21	1.5	73.79	3.5	67.15	2.8
5	3	0.82	4.24	1.91	48.73	1.1	56.65	1.9	69.88	3.1	72.98	3.4
6	2	0.73	3.63	2.19	58	2.0	55.85	1.9	70.83	3.2	63.96	2.6
6	3	0.81	3.38	2.1	55.54	1.7	52.96	1.5	73.45	3.4	65.69	2.7
7	2	0.7	3.98	2.01	60.81	2.2	62.06	2.4	66.51	2.8	60.93	2.3
7	3	0.71	3.73	2.04	58.77	2.0	56.37	1.9	71.19	3.2	60.97	2.3
8	3	0.62	4.04	1.98	61.66	2.3	60.12	2.3	68.83	3.0	59.3	2.2
8	4	0.71	3.18	1.84	61.33	2.3	63.74	2.6	68.07	2.9	60.25	2.2
9	3	0.69	3.83	2.14	62.64	2.4	66.08	2.8	63.53	2.5	60.17	2.2
9	4	0.72	3.15	1.8	63.25	2.5	64.03	2.6	68.73	2.7	61.67	2.4
Average		0.72	3.70	2.01	58.53	2.0	59.01	2.1	69.18	3.0	63.31	2.5

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4A.5: Results of ELECTRE I + Maxscoresum Simulation Runs

					1. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Nash Solution							
					60.61	2.2	66.92	2.9	67.30	2.8	66.36	2.8
Foregr. Set	**C**	**Probability**	**Average Rank**	**Soln. Set Size**	**2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods**							
5	2	0.75	1.54	1	59.69	2.1	63.92	2.7	71.5	3.3	66.85	2.8
5	3	0.75	1.83	1	55.03	1.7	68.97	3.1	67.85	2.9	70.36	3.2
6	2	0.67	1.91	1	60.15	2.2	64.1	2.7	69.41	3.1	66.53	2.8
6	3	0.76	1.45	1	60.37	2.2	65.1	2.8	70.38	3.2	65.01	2.7
7	2	0.6	2	1	60.47	2.3	66.05	2.9	70.24	3.2	64.44	2.7
7	3	0.64	2.26	1	60.78	2.3	65.61	2.8	70.36	3.2	61.12	2.4
8	3	0.7	2.2	1	61.42	2.4	68.77	3.1	68.45	3.0	62.85	2.6
8	4	0.73	1.55	1	62.71	2.5	67.45	3.0	69.43	3.0	61.86	2.4
9	3	0.74	1.45	1	61.09	2.3	64.56	2.8	69.47	3.1	66.04	2.8
9	4	0.74	1.74	1	62.68	2.5	66.51	2.9	68.97	3.0	65	2.7
Average		0.71	1.79	1.00	60.44	2.3	66.10	2.9	69.61	3.1	65.01	2.7

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4A.6: Results of ELECTRE I + Minranksum Simulation Runs

Foregr. Set	C	Probability	Average Rank	Soln. Set Size	1. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Nash Solution							
					60.61	2.2	66.92	2.9	67.30	2.8	66.36	2.8
					2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
5	2	0.61	2.32	1.27	57.51	1.9	60.28	2.4	73.03	3.4	65.83	2.8
5	3	0.66	2.15	1.19	52.02	1.4	69.18	3.1	66.89	2.8	71.6	3.3
6	2	0.58	2.16	1.33	61.99	2.4	58.89	2.2	71.83	3.3	63.02	2.5
6	3	0.73	2.05	1.31	57.81	2.0	63.05	2.5	70.64	3.2	65.67	2.7
7	2	0.54	2.78	1.29	61.71	2.4	58.73	2.3	72.91	3.4	62.27	2.5
7	3	0.43	3.33	1.34	60.62	2.3	54.86	1.9	72.82	3.4	60.18	2.3
8	3	0.5	3.21	1.23	62.02	2.4	61.14	2.5	71.88	3.3	59.44	2.3
8	4	0.53	2.68	1.2	63.1	2.5	65.34	2.8	69.76	3.1	58.47	2.2
9	3	0.55	2.87	1.21	63.92	2.6	61.81	2.6	70.17	3.2	59.88	2.3
9	4	0.56	2.44	1.16	64.61	2.6	62.14	2.6	70.31	3.2	60.77	2.4
Average		0.57	2.60	1.25	60.53	2.3	61.54	2.5	71.02	3.2	62.71	2.5

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4A.7: Results of ELECTRE I + Max/Min Simulation Runs

					1. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Nash Solution							
					60.61	2.2	66.92	2.9	67.30	2.8	66.36	2.8
Foregr. Set	**C**	**Probability**	**Average Rank**	**Soln. Set Size**	**2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods**							
5	2	0.44	2.22	1	52.5	1.4	61.78	2.4	69.68	3.1	70.28	3.2
5	3	0.5	1.96	1	50.81	1.2	63.26	2.5	65.96	2.7	73.43	3.4
6	2	0.55	1.95	1	57.55	1.9	59.4	2.3	70.11	3.1	65.67	2.7
6	3	0.65	1.73	1	55.85	1.7	61.59	2.4	69.77	3.1	67.84	2.9
7	2	0.58	1.78	1	59.17	2.1	63.33	2.6	67.5	2.9	64.77	2.7
7	3	0.65	1.84	1	58.81	2.0	57.37	2.0	70.88	3.2	64.22	2.6
8	3	0.64	1.52	1	59.34	2.1	64.25	2.6	65.46	2.7	65.32	2.7
8	4	0.68	1.57	1	57.83	1.9	63.77	2.6	67.48	2.8	65.65	2.7
9	3	0.7	1.56	1	59.95	2.1	62.32	2.5	67.56	2.9	66.26	2.8
9	4	0.66	1.58	1	59.89	2.1	64.22	2.6	66.95	2.8	65.21	2.7
Average		0.61	1.77	1.00	57.17	1.9	62.13	2.5	68.14	2.9	66.87	2.8

* in the order: forestry, irrigation, rural communities and conservation

APPENDIX 4B: THE DETAILED RESULTS OF THE SENSITIVITY ANALYSIS CONDUCTED ON THE VARIOUS MCDM APPROACHES FOR THE FIRST DATA SET

In this appendix we provide the detailed tabulated outcomes of the sensitivity analysis conducted on the various MCDM approaches for the first data set. The tables listed below have column headings that consist of shortened descriptions. Their full descriptions are provided at the beginning of appendix 4A. The explanation of the symbols α and σ has also previously been provided in section 4.1 of this chapter.

The tables for the sensitivity analysis conducted on the various combinations of MCDM methods are listed on the next few pages.

TABLE 4B.1: Results of Maxscoresum(σ) + Maxscoresum(σ) Simulation Runs

σ	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
2	3	0.78	1.55	1	63	2.5	66.93	2.9	67.93	2.9	65.89	2.8
5	3	0.69	2.21	1	63.09	2.5	68.45	3.1	69.27	3.0	60.29	2.4
10	3	0.59	2.22	1	60.23	2.2	61.83	2.5	70	3.1	65.78	2.8
2	4	0.65	1.89	1	63.92	2.6	66.92	3.0	68.12	3.0	62.75	2.6
5	4	0.72	2.07	1	63.88	2.6	65.71	2.9	67.86	2.9	63.41	2.6
10	4	0.48	2.88	1	63.89	2.5	63.71	2.8	67.89	3.0	59.65	2.3

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.2: Results of Minranksum(σ) + Minranksum(σ) Simulation Runs

σ	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
2	3	0.58	2.73	1.44	63.29	2.5	60.06	2.4	71.39	3.3	59.26	2.1
5	3	0.55	3.49	1.41	62.3	2.4	61.72	2.6	69.34	3.1	60.2	2.3
10	3	0.58	2.67	1.29	62.45	2.4	62.48	2.6	70.01	3.2	61.33	2.4
2	4	0.57	3.13	1.28	64.02	2.6	61.83	2.5	67.34	6.9	59.69	2.2
5	4	0.56	3.03	1.18	62.95	2.5	64.24	2.8	68.76	3.0	61.46	2.4
10	4	0.51	3.11	1.18	59.44	2.2	63.04	2.7	69.48	3.1	63.02	2.6

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.3: Results of Max/Min(σ) + Max/Min(σ) Simulation Runs

σ	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
2	3	0.64	1.53	1	57.47	1.9	63.77	2.6	66.03	2.7	66.24	2.8
5	3	0.53	1.89	1	59.11	2.1	62.15	2.4	67.39	2.8	63.34	2.5
10	3	0.65	1.75	1	57.64	1.9	62.97	2.5	65.3	2.6	66.49	2.8
2	4	0.6	1.76	1	59.39	2.1	60.89	2.3	66.62	2.8	66.24	2.7
5	4	0.55	1.89	1	59.22	2.0	59.98	2.2	65.96	2.7	64.43	2.6
10	4	0.53	2.12	1	56.83	1.8	60.52	2.3	66.81	2.8	66.29	2.8

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.4: Results of ELECTRE I(σ) + ELECTRE I(σ) Simulation Runs

σ	C	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
2	2	0.71	4.12	2.01	61.47	2.3	61.12	2.3	68.41	3.0	59.93	2.2
5	2	0.64	4.34	1.75	59.4	2.1	61.14	2.4	68.65	3.0	62.04	2.5
10	2	0.58	4.30	1.75	59.88	2.2	60.47	2.3	65.09	2.7	62.31	2.5
2	3	0.75	3.34	1.86	61.69	2.3	59.72	2.2	69.53	3.1	62.51	2.4
5	3	0.78	3.46	2.01	58.69	2.0	58.06	2.1	69.38	3.1	63.29	2.5
10	3	0.62	4.36	2.11	58.64	2.0	53.91	1.7	72.03	3.3	60.69	2.3

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.5: Results of ELECTRE I(σ) + Maxscoresum(σ) Simulation Runs

σ	C	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
2	2	0.65	2.3	1	62.37	2.4	68.28	3.1	68.22	3.0	61.9	2.5
5	2	0.62	2.62	1	62.14	2.4	67.78	3.0	67.92	2.9	63.72	2.7
10	2	0.55	2.8	1	61.33	2.4	61.14	2.5	69.18	3.1	63.63	2.6
2	3	0.76	1.66	1	61.27	2.3	64.67	2.8	70.42	3.2	67	2.9
5	3	0.74	2.06	1	61.01	2.3	65.67	2.9	68.82	3.0	64.5	2.7
10	3	0.54	2.17	1	62.16	2.4	61.54	2.5	70.44	3.2	61.83	2.5

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.6: Results of ELECTRE I(σ) + Minranksum(σ) Simulation Runs

σ	C	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
2	2	0.5	3.41	1.26	63.45	2.5	60.32	2.5	70.76	3.2	59.17	2.3
5	2	0.56	3.67	1.36	59.83	2.2	62.35	2.6	69.72	3.1	63.06	2.6
10	2	0.5	3.83	1.24	58.24	2.1	56.21	2.0	70.42	3.2	65.2	2.8
2	3	0.63	2.32	1.22	60.78	2.2	59.01	2.3	72.11	3.4	66.16	2.8
5	3	0.62	2.53	1.25	60.05	2.2	61.13	2.5	70.62	3.2	62.56	2.5
10	3	0.44	3.27	1.29	59.94	2.2	57.06	2.1	71.11	3.2	61.91	2.5

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.7: Results of ELECTRE I(σ) + Max/Min(σ) Simulation Runs

σ	C	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
2	2	0.63	1.74	1	61.55	2.3	63.95	2.6	66.59	2.8	62.74	2.4
5	2	0.58	2.14	1	60.16	2.2	62.97	2.6	67.99	2.9	63.59	2.6
10	2	0.49	2.35	1	58.52	2.0	57.12	2.0	67.54	2.9	65.61	2.7
2	3	0.59	1.58	1	58.21	2.0	60.8	2.4	70.09	3.1	68.17	3.0
5	3	0.48	2.17	1	55.67	1.7	59.75	2.3	68.07	2.9	66.14	2.8
10	3	0.54	2.25	1	57.17	1.9	56.25	2.0	70.85	3.2	65.11	2.7

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.8: Results of ELECTRE I(α) + ELECTRE I(α) Simulation Runs

α	C	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
0.05	2	0.65	4.99	2.71	57.72	1.9	55.2	1.8	68.51	3.0	60.26	2.2
0.15	2	0.72	5.24	2.43	60.85	2.2	65.05	2.7	62.9	2.4	56.73	1.8
0.25	2	0.58	6.08	2.89	51.53	1.3	53.85	1.6	62.64	2.4	67.16	2.9
0.40	2	0.59	6.11	2.69	51.27	1.3	53.63	1.6	63.05	2.4	67.7	3.0
0.05	3	0.81	3.95	2.57	60.03	2.1	54.3	1.6	70.49	3.1	60.31	2.2
0.15	3	0.87	4.16	2.6	62.39	2.3	54.41	1.6	71.38	3.2	58.69	2.0
0.25	3	0.76	4.34	2.32	65.91	2.7	56.16	1.8	69.93	3.1	54.01	1.6
0.40	3	0.76	4.42	2.6	58.45	2.0	59.28	2.2	67.99	2.9	60.58	2.2

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.9: Results of ELECTRE I(α) + Maxscoresum Simulation Runs

α	C	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
0.05	2	0.65	2.02	1	63.09	2.5	66.03	2.9	70.5	3.2	59.24	2.2
0.15	2	0.77	1.9	1	62.01	2.4	68.1	3.1	69.84	3.1	62.61	2.5
0.25	2	0.59	2.27	1	62.26	2.4	63.15	2.7	70.2	3.2	61.35	2.4
0.40	2	0.65	2.12	1	61.38	2.3	63.43	2.7	69.15	3.1	63.67	2.7
0.05	3	0.76	1.64	1	60.14	2.2	66.81	2.9	67.93	2.9	66.41	2.9
0.15	3	0.83	1.6	1	63.44	2.5	66.45	2.9	69.79	3.1	64.97	2.8
0.25	3	0.84	1.6	1	63.35	2.5	67.45	2.9	69.16	3.0	64.83	2.7
0.40	3	0.76	1.43	1	61.21	2.3	68.31	3.0	69.57	3.1	64.23	2.6

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.10: Results of ELECTRE I(α) + Minranksum Simulation Runs

α	C	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
0.05	2	0.45	2.89	1.27	62.92	2.5	56.45	2.1	73.76	3.5	57.54	2.0
0.15	2	0.64	3.11	1.42	61.17	2.3	63.31	2.7	71.16	3.2	59.78	2.3
0.25	2	0.48	2.79	1.31	61.12	2.3	59.53	2.3	70.6	3.2	61.16	2.4
0.40	2	0.5	3.27	1.38	59.77	2.2	58.35	2.2	69.89	3.1	61.3	2.4
0.05	3	0.69	2.38	1.34	61.49	2.3	63.48	2.7	69.79	3.1	62.1	2.5
0.15	3	0.67	2.48	1.23	61.72	2.4	63.19	2.6	71.34	3.3	63.28	2.6
0.25	3	0.66	2.49	1.32	61.28	2.3	62.75	2.6	72.44	3.4	62.76	2.5
0.40	3	0.65	2.60	1.35	60.91	2.3	64.81	2.8	70.89	3.2	61.93	2.4

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.11: Results of ELECTRE I(α) + Max/Min Simulation Runs

α	C	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
0.05	2	0.6	1.79	1	59.7	2.1	61.39	2.5	70.48	3.2	60.44	2.2
0.15	2	0.71	1.66	1	59.82	2.1	63.69	2.7	69.92	3.1	62.94	2.4
0.25	2	0.45	2.28	1	57.93	2.0	59.85	2.3	66.86	2.8	64.08	2.6
0.40	2	0.62	1.78	1	57.02	1.9	59.88	2.2	67.11	2.8	66.83	2.8
0.05	3	0.59	1.78	1	56.84	1.8	60.85	2.3	69.23	3.0	65.86	2.8
0.15	3	0.59	1.69	1	58.66	2.0	61.61	2.4	70.43	3.1	65.24	2.7
0.25	3	0.67	1.68	1	59.34	2.1	61.43	2.3	70.44	3.1	66.75	2.8
0.40	3	0.69	1.62	1	57.94	1.9	65.57	2.7	67.02	2.8	65.59	2.7

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.12: Results of ELECTRE I(α)(σ) + ELECTRE I(α)(σ) Simulation Runs

α	σ	C	Prob.	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
0.4	10	2	0.47	7.15	2.76	43.25	1.5	48.4	1.03	61.7	2.3	77.22	3.8
0.4	10	3	0.63	5.08	2.42	54.08	1.5	59.56	2.2	66.75	2.8	64.13	2.6

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.13: Results of ELECTRE I(α)(σ) + Maxscoresum(σ) Simulation Runs

α	σ	C	Prob.	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
0.4	10	2	0.52	2.5	1	58.53	2.1	59.73	2.3	68.62	3.0	67.6	3.0
0.4	10	3	0.6	2.14	1	57.96	2.0	63.82	2.7	69.75	3.1	68.01	3.0

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.14: Results of ELECTRE I(α)(σ) + Minranksum(σ) Simulation Runs

α	σ	C	Prob.	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
0.4	10	2	0.46	3.46	1.33	57.73	2	59.02	2.2	66.77	2.9	66.05	2.9
0.4	10	3	0.59	2.55	1.33	57.99	2.0	59.77	2.3	70.63	3.2	66.99	2.9

* in the order: forestry, irrigation, rural communities and conservation

TABLE 4B.15: Results of ELECTRE I(α)(σ) + Max/Min(σ) Simulation Runs

α	σ	C	Prob.	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
0.4	10	2	0.46	2.76	1	54.2	1.6	53.76	1.7	68.39	3.0	71.09	3.3
0.4	10	3	0.5	2.11	1	55.89	1.7	60.01	2.3	67.65	2.9	68.35	3.0

* in the order: forestry, irrigation, rural communities and conservation

CHAPTER 5: ANALYSIS OF THE SECOND DATA SET

5.1 Procedure for Implementing the MCDM methods

We formulated the randomly generated data set in such a way that it would closely resemble the previously analyzed (or first) data set. Our main reason for doing this is that we wanted to test whether the results obtained from the first data set would be repeated in the randomly generated data set. Furthermore, we were also interested in obtaining any new results as well as exploring the effects that external factors have on the results produced by the various MCDM approaches. For the analysis on this data set, we started with $p = 4$ interest groups, $m = 4$ policy elements generated by means of the experimental design procedure described in section 3.3 of chapter 3, and $n = 4$ derived attributes generated from these policy elements. The effects on the compromise solution that result when we change these three external factors are reported in section 5.4, where we vary the values for p, m and n.

In order to further simulate real life decision events, we also re–generated the 4 derived attributes from one iteration to the next. We previously described in section 3.3 of chapter 3, how we varied the attributes (forming the surrogate planning objectives for the various interest groups) from one iteration to the next for a particular simulation run.

The MCDM methods were therefore employed at the labelled stages of the simulation algorithm (figure 3.1 of chapter 3), for a particular simulation run consisting of 100 iterations, in the combinations shown below.

Stage 1 Method	Stage 2 Method
Maxscoresum	Maxscoresum
Minranksum	Minranksum
Max/Min	Max/Min
ELECTRE I	ELECTRE I

The efficiency of these MCDM methods would again be represented by the quality of the final solution produced by each combination of methods used.

In order to confirm some of the results obtained from the first data set, the sensitivity analysis was again conducted by looking at possible errors on the thermometer scores (for all methods) and the concordance weights (for the ELECTRE I method specifically) for the various MCDM approaches.

When perturbing the interest thermometer scores, the MCDM methods were combined as shown below.

Stage 1 Method	Stage 2 Method
Maxscoresum(σ)	Maxscoresum(σ)
Minranksum(σ)	Minranksum(σ)
Max/Min(σ)	Max/Min(σ)
ELECTRE I(σ)	ELECTRE I(σ)

The symbol σ (sigma), shown in brackets next to each method, was previously explained in section 4.1 of chapter 4. We will provide the exact figures used for σ when reporting the results of the simulation runs for the sensitivity analysis in appendix 5B at the end of this chapter.

The second area of the sensitivity analysis was that of perturbing the concordance weights that were used in the ELECTRE I method. The ELECTRE I method was combined as follows:

Stage 1 Method	Stage 2 Method
ELECTRE I(α)	ELECTRE I(α)

We will again the exact figures used for α (alpha), previously explained in section 4.1 of chapter 4, in appendix 5B at the end of this chapter.

In order to investigate any cumulative effects for the potential sources of error, we combined the area of perturbing concordance weight assignments with that of perturbing thermometer scores in one set of sensitivity runs. The manner in which the ELECTRE I method was combined in order to complete this part of the sensitivity analysis is shown below.

Stage 1 Method	Stage 2 Method
ELECTRE I(α)(σ)	ELECTRE I(α)(σ)

We will again provide the exact figures used for α and σ in appendix 5B, at the end of this chapter.

5.2 Results Produced by the MCDM Methods

The results that provide the detailed outcome of the simulation runs conducted for the various MCDM approaches, are tabled in appendix 5A at the end of this chapter. The description of these results for the particular combination of MCDM methods is given below, with each description again containing a reference (in its heading) to the table number in appendix 5A to which it refers.

When doing the analysis on this randomly generated data set, we found that the average utility values of the Nash solution did not favour any particular interest group and a fair spread of the utility values across all interests occurred. This was also the case for the average utility values of the compromise solution for all the combinations of MCDM methods. We also found that the Nash values for the Background Set scenarios were all more closely clustered when compared to the first data set. This was a result of the good spread of utility values that occurred for this data set. This would imply that the Nash optimum would have one or two other alternatives (ranked second and third best) that had Nash values very close to that of the Nash optimum, and that they were just as likely to be viewed as good "benchmark" alternatives.

Maxscoresum + Maxscoresum (table 5A.1)
This combination of MCDM methods provides us with a probability of selecting the Nash optimum as our solution, that is on average equal to 63%, compared to 72% for the previous data set. This discrepancy is explained by the fact that the Nash values are more closely clustered for this data set, and the effect hereof was explained in the paragraph above. We will also find that this pattern in the results will be repeated for the remaining three combinations of methods. The solution consists of a single best policy scenario that has an average ranking equal to 3.51. Furthermore, the utility values of the respective interest groups for our best compromise solution, compare favourably to those of the Nash optimum (provided at the top of the table), which was used as a basis for comparison. The average utility values for the compromise solution (at the bottom of the table), tend to lie within 1 unit (on the 0–100 scale) of the Nash optimum, and they also represent a good compromise solution to all groups, centring around 76 units in utility preference.

There is no marked effect in the results when either changing the value for K, i.e. the number of scenarios discarded when revising the Foreground Set, or increasing the size of the Foreground Set.

Minranksum + Minranksum (table 5A.2)

The probability of selecting the Nash optimum as an element of our solution set (consisting on average of 1.37 policy scenarios) is on average 59%. This compares to the figure of 56% obtained for the previous data set. The scenarios in our solution set have an average ranking of 4.17, whereas for the first data set we had an average ranking equal to 3.13. We also find that the average utility values for the best solution are on average marginally lower than those of the Nash solution, and tend to lie within 1 to 2 units (on the 0–100 scale) of the Nash optimum.

There are no noticeable effects in the results when changing either the size of the Foreground Set or the number of scenarios discarded (K) whilst revising the Foreground Set.

Max/Min + Max/Min (table 5A.3)

The solution produced by this combination of MCDM methods consists of a single policy scenario and there is on average a 47% probability of selecting the Nash optimum as this solution (compared to 61% before). This policy scenario has an average ranking equal to 2.67, whereas for the previous data set it had an average ranking in table 4A.3 equal to 1.72. The average utility values for the four interest groups are spread evenly amongst these interests but are lower than the average utilities for the Nash optimum solution. The compromise solution has utilities fluctuating around 71 units whilst the Nash optimum has utilities around 76 units.

There are also no noticeable effects in the results when changing either the size of the Foreground Set or the number of scenarios discarded (K) when revising the Foreground Set.

ELECTRE I + ELECTRE I (table 5A.4)

In a solution set that consists, on average, of 1.86 alternatives, we have a 68% chance of selecting the Nash optimum as an element of the set (72% was the probability for the previous data set). The scenarios in our solution tend to have an average ranking of 5.66, when we had an average ranking of 3.70 for the previous data set. We do find that the average utility values for the compromise solution are lower than those of the Nash solution. The latter has utilities fluctuating around 76 units whilst the values for the compromise solution tend to centre around 73 units.

When considering the probability of selecting the Nash optimum as part of the solution set, one can see that, with the exception of a Foreground Set of size 9 this time (it was a set of size 7 before), this statistic increases as the value for C increases. In other words, by eliminating less or retaining more scenarios when revising the Foreground Set, the greater our chance becomes of including the

Nash optimum in our solution. There are no marked changes in the results when looking at the different sizes for the Foreground Sets.

5.3 Sensitivity Analysis

In the sensitivity analysis we would first perturb the interest thermometer scores and follow this by perturbing the concordance weights assigned to the various interest groups. We would then combine both sources of error in the same set of simulation runs.

It must be remembered that we conducted the sensitivity runs on the different combinations of methods using Foreground Sets of size 7 policy scenarios only. The results for these sensitivity runs are tabulated in appendix 5B at the end of this chapter. We again include the table number reference (for the appendix 5B) in the heading of the description of the results for each combination of methods.

5.3.1 Perturbing the Interest Thermometer Scores

Maxscoresum(σ) + Maxscoresum(σ) (table 5B.1)

For K = 3, there is a deterioration of the results when compared to the corresponding ones in table 5A.1. This is seen mainly in a decrease of the probability of selecting the Nash optimum when σ changes in value from 2 (0.61) to 5 (0.5). For K = 4, the probability of selecting the Nash optimum deteriorates more when σ changes from 5 (0.63) to 10 (0.51). There is also a marked worsening of the results when one examines the average ranking of the solution and compare this to the results obtained from table 5A.1.

We also find that the utility values for the compromise solution tend to lie within 2 to 6 units (on the 0–100 scale) or 3 to 8% of the Nash optimum. This is more than the 1 unit recorded for the unperturbed results in table 5A.1, and would seem to suggest that this combination of MCDM methods is not robust when the interest thermometer scores are subjected to certain levels of precision error. We obtained a similar result from the previous data set.

Minranksum(σ) + Minranksum(σ) (table 5B.2)

There are marked changes in these results when compared to the unperturbed results obtained in table 5A.2. These changes, especially amplified when K = 4 and σ = 10, occur for both the probability of selecting the Nash optimum as well as the average ranking of scenarios in the solution set.

Furthermore, the utility values for the compromise solution tend to lie within 3 to 8 units (on the 0–100 scale) or 4 to 11% of the Nash optimum. This is more than the 1 to 2 units recorded for the unperturbed results in table 5A.2, and this combination of MCDM methods is therefore not robust to precision errors being made on interest thermometer scores. We did not obtain the same result for the analysis conducted on the previous data set.

Max/Min(σ) + Max/Min(σ) (table 5B.3)
There were noticeable changes in the results when the thermometer scores were perturbed, compared to the unperturbed results in table 5A.3. The deterioration of the results are visible when looking at the probability of selecting the Nash optimum for both K = 3 and 4, with changes being more noticeable for K = 4.

However, the utility values for the compromise solution tend to lie within 7 to 10 units (on the 0–100 scale) or 9 to 13% of the Nash optimum. This is only marginally higher than the 4 to 6 units (or 5 to 8%) recorded for the unperturbed results in table 5A.3. Therefore, on the whole, it would seem that this combination of methods is robust to precision errors being made on the thermometer scores. We obtained a similar result for the analysis conducted on the previous data set.

ELECTRE I(σ) + ELECTRE I(σ) (table 5B.4)
The probability of selecting the Nash optimum was not affected negatively when this combination of MCDM methods was exposed to a precision error being made on the thermometer scores. The average ranking of the scenarios in the compromise solution set deteriorated most notably for σ = 10, when compared to the unperturbed set of results tabulated in table 5A.4. This deterioration occurred for both C = 2 and 3 in a Foreground Set consisting of 7 scenarios.

Furthermore, the utility values for the compromise solution tend to lie within 4 to 9 units (on the 0–100 scale) or 5 to 12% of the Nash optimum. This is more than the 3 units recorded for the unperturbed results in table 5A.4, and this combination of MCDM methods is therefore not robust to precision errors being made on the interest thermometer scores. For the analysis on the previous data set, we obtained a similar result.

5.3.2 Perturbing the Concordance Weights

The second part of the sensitivity analysis looked at perturbing the concordance weights, used in the ELECTRE I method, that were assigned to each interest group.

ELECTRE I(α) + ELECTRE I(α) (table 5B.5)

For C = 2, there is a deterioration in the average ranking of the solution set scenarios from the first level of α introduced. The unperturbed value for this statistic was 5.53 in table 5A.4, and we see that at α = 0.05 it already has a value of 6.82. This deteriorates further to reach a value of 6.98 for α = 0.15. There is, however, no deterioration of the probabilities to select the Nash optimum, as was the case for the previous data set, for both C = 2 and 3. The average ranking of the solution set scenarios, however, worsens for C = 3 from the first level of proportional error introduced.

Furthermore, the utility values for the compromise solution tend to lie within 10 to 15 units (on the 0–100 scale) or 13 to 20% of the Nash optimum. This is substantially more than the 3 units recorded for the unperturbed results in table 5A.4, and this combination of MCDM methods therefore appears not to be robust to proportional errors being made on the concordance weights. This was also the case for the analysis conducted on the previous data set.

5.3.3 Perturbing both the Interest Thermometer Scores and the Concordance Weights

When perturbing both the interest thermometer scores and the concordance weights simultaneously, we were interested to see whether these sources of error affected the outcome in an additive manner, or whether some form of interaction had occurred.

ELECTRE I(α)(σ) + ELECTRE I(α)(σ) (table 5B.6)

The results obtained from the sensitivity runs conducted for this combination of methods deteriorate when one looks at the average ranking of the scenarios in the solution set in particular. There appears to be an additive effect causing this further deterioration to occur when compared to the results obtained when the combination of methods was exposed to both sources of error separately.

The utility values for the compromise solution tend to lie within 8 to 13 units (on the 0–100 scale) or 11 to 17% of the Nash optimum. This is markedly more than the 3 units recorded for the unperturbed results in table 5A.4, and would seem to suggest that this combination of MCDM methods is not robust when exposed to both sources of error simultaneously.

5.4 Effects of External Factors

The effects that the following three external factors have on the compromise solution produced by the various combinations of MCDM methods was investigated. These factors are (i) the number of policy elements, (ii) the number of derived attributes, and (iii) the number of interest groups. The examination of the effects of these external factors was conducted on the different combinations of methods using Foreground Sets of size 7 scenarios only (again as motivated by the work of Miller: 1956). The effect of each external factor was investigated separately for the combinations of MCDM methods shown below.

Stage 1 Method	**Stage 2 Method**
Maxscoresum	Maxscoresum
Minranksum	Minranksum
Max/Min	Max/Min
ELECTRE I	ELECTRE I

The tables providing the detailed outcome of these investigations are provided in appendix 5C at the end of this chapter. We describe the effects of each external factor on the particular combination of methods below and again include in the heading of each description, the reference to the table number in appendix 5C to which it refers.

5.4.1 Number of Policy Elements

Maxscoresum + Maxscoresum (table 5C.1)
For K = 3, we see that when 6 policy elements were introduced, the probability of selecting the Nash optimum decreased to 50%. The average ranking of solution set scenarios also worsened and they had an average ranking equal to 5.68. For K = 4, the average ranking of the solution increased to 4.75 and 4.08 when 5 and 6 policy elements were introduced respectively.

The above results suggest that, on the whole, there is not a marked effect on the probability of selecting the Nash optimum when more policy elements are introduced. There does, however, appear to be some deterioration in the average ranking of the solution as more policy elements are introduced. If we were to look at the average value for this statistic (taken over all Foreground Set sizes and values for K) reflected in table 5A.1, we will see that it is 3.51. This value, when used as a benchmark, further indicates that there is some deterioration in the value of this statistic as the number of policy elements increased. This is particularly noticeable when K = 3 and 6 policy elements were used in the simulation runs for this combination of MCDM methods.

However, the utility values for the compromise solution tend to lie within 3 to 5 units (on the 0–100 scale) of the Nash optimum. This compares favourably with the 1 unit recorded for the unperturbed results in table 5A.1. It is therefore not clear as to whether the results deteriorated that noticeably as the number of policy elements included in the study increased for this combination of MCDM methods.

Minranksum + Minranksum (table 5C.2)
For both K = 3 and 4 there is a deterioration in the probability of selecting the Nash optimum as more policy elements are introduced. There is also a marked increase in the average ranking of scenarios in the solution set as more policy elements are included in the simulation runs for this combination of methods. However, the deterioration for this statistic is less pronounced for K = 4 when one takes into account this statistic's average value (taken over all set sizes and values for K in table 5A.2) of 4.17. This value indicates that for K = 4, the results only deteriorate markedly when 6 policy elements are introduced.

The utility values for the compromise solution tend to lie within 3 to 7 units (on the 0–100 scale) of the Nash optimum. This is more than the 1 to 2 units recorded for the unperturbed results in table 5A.2. Overall, these results therefore tell us that this combination of MCDM methods produces progressively worse solutions as the number of policy elements are increased in the study.

Max/Min + Max/Min (table 5C.3)
The probability of selecting the Nash optimum deteriorates for both K = 3 and 4 as the number of policy elements used in the study increase. The average ranking of the solution also increases as the policy elements used increase in number. This, however, is more

noticeable when the policy elements increase from 5 to 6 for K = 4. For both the previous mentioned statistics, we find that the deterioration of the results are less obvious when we compare the values for these statistics to their respective averages (taken over all set sizes and values for K) shown in table 5A.3. These average value are 47% and 2.67 for the probability of selecting the Nash optimum and the average ranking of the solution respectively.

Furthermore, the utility values for the compromise solution tend to lie within 4 to 6 units (on the 0–100 scale) of the Nash optimum. This is the same as the 4 to 6 units recorded for the unperturbed results in table 5A.3. On the whole, the results produced by this combination of MCDM methods, do not deteriorate noticeably as the number of policy elements included in the study increase.

ELECTRE I + ELECTRE I (table 5C.4)

The probability of selecting the Nash optimum deteriorates for both C = 2 and 3 (more noticeably for C = 3) as the number of policy elements used in the study increase. There is also a very marked increase in the average ranking of solution set scenarios for both cases of C, and this is especially noticeable when 6 policy elements are utilised in the study.

However, the utility values for the compromise solution tend to lie within 5 to 7 units (on the 0–100 scale) of the Nash optimum. This compares favourably with the 3 to 4 units recorded for the unperturbed results in table 5A.4. It is therefore not clear as to whether or not this combination of MCDM methods provides one with noticeably deteriorating results as the number of policy elements used in the study increase in number.

5.4.2 Number of Derived Attributes

Maxscoresum + Maxscoresum (table 5C.5)

For both the probability of selecting the Nash optimum as well as the average ranking of the solution, there are no noticeable changes in the results as the number of derived attributes used in the study increased. The marginal changes that did take place, occurred mainly for K = 4, when we noticed a marginal increase in the probability of selecting the Nash optimum as the solution, as the number of derived attributes increased.

However, this marginal improvement only occurred for the one statistic mentioned above, and we can therefore say that as the number of derived attributes increased, there were no marked effects reflected in the results, where this combination of MCDM methods is concerned. The utility values for the compromise solution further support this conclusion, since they tend to lie within 3 to 5 units (on the 0–100 scale) of the Nash optimum. This compares favourably with the 1 unit recorded for the unperturbed results in table 5A.1.

Minranksum + Minranksum (table 5C.6)
For both K = 3 and 4 there is a deterioration of the results, as measured by the probability of selecting the Nash optimum, when 5 derived attributes are used in the study. However, when 6 derived attributes are used, the probability improves noticeably for both K = 3 and 4. For K = 3, we also see that the average ranking of the solution set scenarios improves marginally. However, for K = 4, this statistic deteriorates noticeably.

The utility values for the compromise solution tend to lie within 3 to 6 units (on the 0–100 scale) of the Nash optimum. This compares favourably with the 1 to 2 units recorded for the unperturbed results in table 5A.2, and it is therefore not very clear as to what pattern the results follow (as more derived attributes are used in the study) for this combination of MCDM methods.

Max/Min + Max/Min (table 5C.7)
For both K = 3 and 4, and where 5 derived attributes are used in the study, the probability of selecting the Nash optimum as the solution and the average ranking of the solution deteriorates. This deterioration continues when 6 derived attributes are included in the study for K = 3, but an improvement takes place in the results for K = 4.

Furthermore, the utility values for the compromise tend to lie within 5 to 7 units (on the 0–100 scale) of the Nash optimum, and this compares favourably with the 4 to 6 units recorded for the unperturbed results in table 5A.3. Where this combination of MCDM methods is concerned, we therefore do not have a clear indication of how the results would be affected as the number of derived attributes used in the study increase.

ELECTRE I + ELECTRE I (table 5C.8)
For both C = 2 and 3, the general trend is for an increase to occur in the probability of selecting the Nash optimum as the solution, as more derived attributes are included in the

study. This trend is more marked in the results for C = 3. There is, however, a deterioration in the average ranking of solution set scenarios for both cases of C, which is more noticeable when 5 derived attributes are used in the study for C = 3.

The utility values for the compromise solution tend to lie within 6 to 9 units (on the 0–100 scale) of the Nash optimum. This is more than the 3 to 4 units recorded for the unperturbed results in table 5A.4, and for this combination of MCDM methods, the results do not provide any definite indications as to whether the increase in the number of derived attributes produces better or worse results.

5.4.3 Number of Interest Groups

The spread of utility values over all the conflicting interest groups remained fairly even, as the number of interests in the simulation study increased, for all (4) combinations of MCDM methods. However, the results indicate that as the number of interest groups increased, so the utility scores (on the 0–100 scale) at which the compromise solution was established, tended to decrease. This is the case for all the combinations of MCDM methods reported on below, but was more noticeable for the last three combinations, *viz.*, the Minranksum + Minranksum, Max/Min + Max/Min and ELECTRE I + ELECTRE I combinations.

Maxscoresum + Maxscoresum (table 5C.9)
There seems to be a definite deterioration of the results as the interest groups increased in number. This deterioration is reflected in both the probability of selecting the Nash optimum as the solution, and more noticeably in the average ranking of the solution.

This combination of MCDM methods therefore produced results that deteriorated as the number of interest groups used in the study increased.

Minranksum + Minranksum (table 5C.10)
When looking at the probability of selecting the Nash optimum and the average ranking of solution set scenarios, we clearly see that the results become worse as the number of interest groups increase in number. This deterioration is particularly noticeable for K = 4, and it is therefore clear that this combination of MCDM methods produced results that deteriorated as the number of interests used in the simulation study increased.

Max/Min + Max/Min (table 5C.11)

For both K = 3 and 4, it is clear that the probability of selecting the Nash optimum as the solution decreases as the interest groups increase in number. However, when looking at the average value (taken over all Foreground Set sizes and values for K) for this statistic provided in table 5A.3, we see that it has a value of 47%. Using this statistic as a benchmark, we still have a deteriorating solution being produced as the number of interests increase, but to a lesser extent than the 53% and 51% would seem to indicate for K = 3 and 4 respectively. Also, the average ranking of the solution remains largely unaffected by the increase in the number of interest groups.

However, the utility values for the compromise solution tend to lie well below 70 units (on the 0–100 scale) for all the interests, as the interest groups increase in number. It therefore appears that this combination of MCDM methods produced results that deteriorated as the number of interest groups in the study increased.

ELECTRE I + ELECTRE I (table 5C.12)

The probability of selecting the Nash optimum as the solution generally increased for both C = 2 and 3, as the number of interest groups used in the simulation study increased in number. The average ranking of solution set scenarios, however, deteriorated for both cases of C as the number of interest groups increased. This was particularly noticeable when 6 interest groups were included in the study.

Furthermore, the utility values for the compromise solution tend to lie below 70 units (on the 0–100 scale) for all the interests, as the interest groups increase in number. It therefore appears that this combination of MCDM methods also produced results that deteriorated as the number of interest groups in the study increased.

5.5 Overall Conclusions for the Analysis on this Data Set

A summary of the results obtained in sections 5.2 and 5.3 is provided in table 5.1 on the following page. In these two sections we evaluated the different MCDM approaches and also tested how robust they were to the precision and proportional errors that were made on the thermometer scores and the concordance weights respectively.

TABLE 5.1: Summary of Results: Data Set No. 2 (Randomly Generated)

Combination of Methods	Combination Performance as Measured by:		Results for Combination Affected by:		How Robust is the Combination when Exposed to:		
	Probability of Selecting the Nash Optimum	Average Ranking of Solution Set Scenarios	Foreground Set Size	K or C	Precision Error (thermometer scores)	Proportional Error (concordance weights)	Precision and Proportional Error
Maxscoresum + Maxscoresum	63%	3.51	no	no	not robust	n/a	n/a
Minranksum + Minranksum	59%	4.17	no	no	not robust	n/a	n/a
Max/Min + Max/Min	47%	2.67	no	no	robust	n/a	n/a
ELECTRE I + ELECTRE I	68%	5.66	no	yes	not robust	not robust	not robust

The conclusions we can draw from the results obtained in sections 5.2 and 5.3 are the following:

(i) From the point of view of quality of solution that the combination of MCDM methods produces, we would recommend the Maxscoresum + Maxscoresum combination. The Max/Min + Max/Min approach fares rather poorly in the probability of selecting the Nash optimum, whilst the ELECTRE I + ELECTRE I approach fares badly where the average ranking of the solution set scenarios is concerned.

(ii) From the point of view of robustness of the combination of MCDM methods, only the Max/Min + Max/Min approach proved to be robust. However, further investigations are required to establish why the remaining combinations fail to be robust for the two potential sources of error on which they were tested.

(iii) On the whole, there does not appear to be a preferred Foreground Set size or preferred values for K or C according to the results obtained.

(iv) Combining criteria (i) and (ii) above, we therefore tend to favour the Maxscoresum + Maxscoresum and the Max/Min + Max/Min approaches. We would possibly further investigate these two approaches, since the first failed to be robust to precision errors being made on the thermometer scores, whilst the latter has a low probability of selecting the Nash optimum as the compromise solution.

These results are fairly similar (at least for the overall conclusions in (iv)) to those obtained when analysing the first data set. We did acknowledge that the first data set had a rather restricting limitation as regards the fact that the surrogate planning objectives were not varied from one iteration to the next for a particular simulation run. We do, however, find that the Max/Min + Max/Min (or best of the worst) and the Maxscoresum + Maxscoresum MCDM approaches, seem to be the two combinations that stand out when compared on the overall quality (including being robust to precision and proportional errors) of the solution they produce, for both data sets.

Both these combinations tend to produce single best policy scenarios as their solution. They also produce a complete rank ordering of the remaining alternatives in the (revised) Foreground Set. This allows the DM the freedom of choice as to decide whether he or she will settle for a single best alternative or include a (subjective) number of the next best ranked alternatives to consider for further investigation.

In table 5.2 below, we summarize the effects that the external factors have on the solution produced by the various MCDM approaches. From this table we see that the number of interest groups tends to be the external factor that most affects the compromise solution produced by the various MCDM approaches. One can expect that as the interests increase in number, so it becomes increasingly more difficult to obtain consensus at maximum levels of efficiency for all the groups concerned. The results reflected these decreasing utility scores (on the 0–100 scale) for the compromise solution, as more interests were included in the study.

The Max/Min + Max/Min approach was the only combination not affected by increasing the number of policy elements in the study, whilst the Maxscoresum + Maxscoresum approach was the only combination not affected by increasing the number of derived attributes. These were the two MCDM approaches we felt we could recommend, based on the results we obtained for both data sets, although more testing should be carried out on these two combinations.

TABLE 5.2: Effects of External Factors for Data Set No. 2 (Randomly Generated)

Combination of Methods	Results for Combination of Methods Affected by:		
	Number of Policy Elements	Number of Derived Attributes	Number of Interest Groups
Maxscoresum + Maxscoresum	not clear	no	yes
Minranksum + Minranksum	yes	not clear	yes
Max/Min + Max/Min	no	not clear	yes
ELECTRE I + ELECTRE I	not clear	not clear	yes

APPENDIX 5A: THE DETAILED RESULTS OF THE VARIOUS MCDM APPROACHES FOR THE SECOND DATA SET

In this appendix we provide the detailed tabulated outcomes of the various MCDM approaches, as applied to the second (randomly generated) data set. The full descriptions of the shortened column headings for the tables in this appendix, are provided in appendix 4A at the end of chapter 4.

The tables of results for implementing the various combinations of MCDM methods start on the following page.

TABLE 5A.1: Results of Maxscoresum + Maxscoresum Simulation Runs

					1. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Nash Solution							
					75.78	2.0	75.19	2.0	76.34	2.0	75.60	2.0
Foregr. Set	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
5	2	0.61	2.9	1	71.88	2.9	77.65	2.3	77.96	2.5	78.41	2.6
5	3	0.52	4.04	1	76.21	2.5	71.78	3.0	74.52	2.7	77.29	2.3
6	2	0.58	4.05	1	76.43	2.5	75.73	2.5	72.71	2.7	76.4	2.7
6	3	0.62	3.72	1	74.83	2.6	71.79	2.6	76.11	2.5	78.69	2.2
6	4	0.67	4.44	1	70.51	3.1	75.41	2.6	78.95	2.2	79.11	2.4
7	2	0.5	4.49	1	73.07	2.9	75.94	2.8	77.31	2.2	72.57	2.6
7	3	0.61	3.2	1	75.33	2.5	74.22	2.3	78.14	2.7	76.68	2.5
7	4	0.65	2.61	1	80.49	2.4	76.63	2.3	78.94	2.2	75.52	2.4
7	5	0.7	3.45	1	73.19	2.7	70.61	2.7	81.24	2.0	80.16	2.1
8	2	0.58	4.32	1	74.95	2.6	76.2	2.4	75.17	2.6	70.99	2.7
8	3	0.64	3.04	1	77.39	2.6	74.95	2.5	75.52	2.1	73.58	2.6
8	4	0.65	4.24	1	76.19	2.8	76.02	2.8	75.53	2.6	79.72	2.0
8	5	0.67	3.07	1	76.72	2.4	76.37	2.4	74.32	2.8	77.27	2.2
8	6	0.72	2.93	1	79.74	2.2	73.53	2.7	75.81	2.6	77.11	2.2
9	2	0.59	3.65	1	76.86	2.3	78.01	2.3	73.62	2.9	78.81	2.5
9	3	0.62	3.34	1	77.08	2.2	75.21	2.6	75.87	2.4	76.9	2.4
9	4	0.61	3.15	1	73.12	2.4	77.91	2.3	75.79	2.6	74.22	2.5
9	5	0.72	3.11	1	79.87	2.1	74.49	2.6	79.52	2.3	73.41	2.6
9	6	0.7	2.94	1	75.24	2.5	75.4	2.4	75.76	2.6	79.09	2.1
9	7	0.63	3.45	1	74.58	2.5	74.42	2.5	76.26	2.4	74.65	2.7
Average		0.63	3.51	1.00	75.68	2.5	75.11	2.5	76.45	2.5	76.53	2.4

* randomly generated

TABLE 5A.2: Results of Minranksum + Minranksum Simulation Runs

Foregr. Set	K	Probability	Average Rank	Soln. Set Size	1. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Nash Solution							
					75.42	2.0	76.39	2.0	76.15	2.0	75.08	2.0
					2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
5	2	0.53	4.56	1.4	71.28	2.6	75.42	2.3	71	2.4	70.72	2.6
5	3	0.55	3.94	1.22	80.58	2.3	76.88	2.0	71.39	2.8	71.45	2.6
6	2	0.49	5.32	1.4	72.09	2.5	69.12	2.8	72.09	2.8	74.82	2.1
6	3	0.59	4.71	1.28	74.4	2.4	75.99	2.3	74.96	2.6	72.86	2.5
6	4	0.6	4.28	1.27	73.16	2.7	75.09	2.5	74.35	2.6	78.34	2.3
7	2	0.57	4.95	1.34	73.28	2.8	79.62	2.1	79.73	2.3	69.04	2.7
7	3	0.63	4.08	1.19	75.3	2.6	74.85	2.2	70.77	2.4	78.06	2.0
7	4	0.69	1.29	3.84	74.54	2.6	78.61	2.2	71.77	2.4	74.48	2.4
7	5	0.65	4.36	1.15	77.01	2.2	72.37	2.8	73.3	2.6	78.92	2.4
8	2	0.65	4.24	1.19	74.87	2.4	79.06	2.4	79.9	2.3	73.59	2.9
8	3	0.64	3.29	1.25	76.44	2.3	77.52	1.9	73.54	2.7	72.23	2.1
8	4	0.57	4.57	1.24	75.05	2.4	72.91	2.6	74.81	2.5	72.95	2.6
8	5	0.61	3.41	1.23	68.37	2.6	79.51	2.2	72.83	2.5	77.4	2.1
8	6	0.56	5.42	1.3	73.04	2.5	69.76	2.9	75.92	2.5	76.35	2.4
9	2	0.48	4.65	1.11	71.87	2.8	76.75	2.6	77.56	2.6	77.28	2.7
9	3	0.64	3.96	1.19	74.67	2.5	72.37	2.6	81	2.1	75.54	2.4
9	4	0.59	4.67	1.18	73.13	2.6	78.82	2.3	73.57	2.5	71.64	2.5
9	5	0.61	4.09	1.2	75.38	2.4	74.69	2.2	74.63	2.6	74.62	2.4
9	6	0.64	3.39	1.24	75.71	2.1	77.88	2.0	76.92	2.1	76	2.3
9	7	0.59	4.17	1.15	75.32	2.4	75.61	2.4	76.46	2.5	74.75	2.4
Average		0.59	4.17	1.37	74.27	2.5	75.64	2.4	74.83	2.5	74.55	2.4

* randomly generated

TABLE 5A.3: Results of Max/Min + Max/Min Simulation Runs

					1. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Nash Solution							
					76.47	2.0	75.37	2.0	76.55	2.0	75.81	2.0
Foregr. Set	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
5	2	0.42	3.29	1	73.55	2.0	68.74	2.4	75.08	2.0	72.5	2.0
5	3	0.47	2.43	1	72.47	2.1	74.96	1.8	72.1	2.0	72.06	2.0
6	2	0.5	2.36	1	73.47	1.8	73.74	2.0	73.24	1.6	70.25	2.1
6	3	0.41	2.57	1	71.62	1.7	72.14	1.8	70.7	1.8	69.61	2.1
6	4	0.42	2.55	1	71.66	2.0	71.52	2.0	73.66	1.8	72.82	2.0
7	2	0.46	2.46	1	68.85	1.9	69.36	1.8	72.45	1.9	73.5	1.8
7	3	0.53	2.37	1	73.21	1.8	73.68	1.8	71.45	1.9	71.46	2.0
7	4	0.51	2.41	1	72.82	1.9	70.99	1.8	73.69	1.8	72.55	1.8
7	5	0.41	3.16	1	72.43	1.8	69.32	1.9	70.65	1.8	66.2	2.2
8	2	0.5	2.34	1	73.98	2.0	72.66	1.8	75.24	1.8	70.69	1.8
8	3	0.5	2.62	1	69.3	1.8	71.66	1.9	72.34	1.7	72.24	1.7
8	4	0.43	2.72	1	71.37	1.7	70.41	1.7	70.14	1.6	69.69	1.8
8	5	0.48	2.69	1	69.45	1.8	71.83	1.8	71.03	1.9	69.27	1.8
8	6	0.52	2.76	1	74.25	2.0	72.31	1.8	71.5	1.8	71.95	1.8
9	2	0.53	2.61	1	73.2	1.9	71.15	2.0	74.16	1.6	70.42	1.9
9	3	0.46	2.77	1	70.48	2.0	69.41	1.6	71.63	1.7	69.47	1.9
9	4	0.49	2.33	1	73.65	1.7	71.74	1.8	72.63	1.9	70.92	1.7
9	5	0.47	2.61	1	70.31	1.8	71.94	1.9	73.29	2.0	71.02	2.0
9	6	0.44	3.06	1	69.63	1.8	70.01	1.9	70.24	1.7	65.73	1.9
9	7	0.45	3.24	1	72.62	1.9	71.42	1.9	69.15	1.7	69.59	1.8
Average		0.47	2.67	1.00	71.92	1.9	71.45	1.9	72.22	1.8	70.60	1.9

* randomly generated

TABLE 5A.4: Results of ELECTRE I + ELECTRE I Simulation Runs

					1. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Nash Solution							
					76.01	2.0	76.11	2.0	76.02	1.9	76.1	2.0
Foregr. Set	C	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
5	2	0.67	5.72	2.18	72.95	2.1	68.12	1.99	70.89	2.05	71.8	2.11
5	3	0.68	6.51	2.04	73.5	2.1	68.12	2.52	71.54	2.14	75.97	1.82
6	2	0.69	5.58	1.87	77.39	2.0	71.45	2.31	71.28	2.29	74.96	2.12
6	3	0.81	4.85	1.95	72.85	2.0	70.79	2.09	73.43	2.01	77.68	1.64
7	2	0.59	5.53	1.72	74.21	2.4	67.91	2.72	76.15	2.02	71.92	2.33
7	3	0.69	4.93	1.78	78.68	2.0	73.28	2.12	71.08	2.28	74.44	2.34
8	3	0.69	5.97	1.88	70.17	2.4	75.36	2.32	71.81	2.31	75.02	2.26
8	4	0.7	5.88	1.8	72.38	2.2	77.19	2.17	66.64	2.59	73.37	2.21
9	3	0.66	5.86	1.54	66.61	2.8	79.38	2.24	77.19	2.24	78.27	2.15
9	4	0.66	5.74	1.87	69.99	2.5	71.27	2.26	76.63	2.14	73.35	2.33
Average		0.68	5.66	1.86	72.87	2.3	72.29	2.3	72.66	2.2	74.70	2.1

* randomly generated

APPENDIX 5B: THE DETAILED RESULTS OF THE SENSITIVITY ANALYSIS CONDUCTED ON THE VARIOUS MCDM APPROACHES FOR THE SECOND DATA SET

This appendix contains the tables of results for the sensitivity analysis conducted on the various MCDM approaches. The shortened column headings of these tables are explained in appendix 4A at the end of chapter 4, whilst the symbols α (alpha) and σ (sigma) are explained in section 4.1 of chapter 4.

The tables are listed on the following 3 pages.

TABLE 5B.1: Results of Maxscoresum(σ) + Maxscoresum(σ) Simulation Runs

σ	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
2	3	0.61	3.73	1	74.92	2.6	74.45	2.5	76.41	2.2	74.78	2.5
5	3	0.5	4.25	1	74.21	2.5	75.46	2.4	75.47	2.5	71.84	2.8
10	3	0.53	3.78	1	74.8	2.6	73.67	2.6	71.67	2.4	75.91	2.4
2	4	0.64	4.17	1	73.19	2.7	74.83	2.4	73.33	2.3	73.9	2.8
5	4	0.63	3.56	1	74.55	2.6	75.99	2.5	75.41	2.5	74.45	2.6
10	4	0.51	4.61	1	73.59	2.6	72	2.6	77.02	2.4	69.37	2.9

* randomly generated

TABLE 5B.2: Results of Minranksum(σ) + Minranksum(σ) Simulation Runs

σ	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
2	3	0.59	5.15	1.27	69.58	2.9	71.21	2.9	74.83	2.3	77.97	2.0
5	3	0.6	4.15	1.21	75.67	2.4	73.94	2.3	76.22	2.1	74.75	2.6
10	3	0.59	4.34	1.2	68.43	2.9	77.33	2.3	71.9	2.3	76.41	2.0
2	4	0.53	4.29	1.2	69.53	2.6	76.19	2.2	74.94	2.5	73.62	2.5
5	4	0.63	3.31	1.29	75.12	2.2	72.51	2.5	72.44	2.2	74.3	2.3
10	4	0.47	5.60	1.22	71.73	2.5	68.86	2.3	74.65	2.3	71.92	2.6

* randomly generated

TABLE 5B.3: Results of Max/Min(σ) + Max/Min(σ) Simulation Runs

σ	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
2	3	0.43	2.47	1	72.14	1.6	70.61	1.8	68.47	1.9	72.59	1.7
5	3	0.41	2.73	1	68.69	1.7	69.71	1.6	69.04	1.9	69.01	1.9
10	3	0.4	2.73	1	69.54	2.0	72.07	2.0	67.51	2.0	72.44	1.8
2	4	0.33	2.94	1	67.48	1.7	68.95	1.6	67.68	1.8	69.58	1.7
5	4	0.36	2.85	1	67.57	1.6	68.83	1.6	67.87	1.8	69.52	1.6
10	4	0.35	3.28	1	66.06	1.8	67.31	1.8	71.42	1.8	68.84	1.8

* randomly generated

TABLE 5B.4: Results of ELECTRE I(σ) + ELECTRE I(σ) Simulation Runs

σ	C	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
2	2	0.68	5.93	1.8	69.43	2.2	71.12	2.4	70.5	2.1	73.29	2.3
5	2	0.66	5.85	1.81	69.1	2.5	71.11	2.3	73.87	2.3	74.03	2.2
10	2	0.6	6.01	1.79	74.93	2.2	68.41	2.6	73.27	2.4	72.37	2.3
2	3	0.68	6.47	1.9	72.41	2.3	69.49	2.6	72.06	2.3	73.43	2.3
5	3	0.71	5.31	1.68	71.51	2.2	72.01	2.3	72.2	2.4	77.02	2.2
10	3	0.63	6.08	1.79	73.23	2.2	67.79	2.4	69.49	2.4	73.61	2.1

* randomly generated

TABLE 5B.5: Results of ELECTRE I(α) + ELECTRE I(α) Simulation Runs

α	C	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
0.05	2	0.68	6.82	2.47	73.02	1.8	62.28	2.2	74.76	1.9	69.68	2.1
0.15	2	0.7	6.98	2.36	72.81	1.9	61.03	2.1	76.31	1.8	66.24	2.2
0.25	2	0.6	6.75	2.44	61.66	2.1	67.94	1.8	83.44	1.6	63.81	1.9
0.40	2	0.64	6.48	2.42	69.75	2.1	72.54	2.0	66.28	2.1	69.15	2.1
0.05	3	0.7	6.88	2.49	68.06	2.3	71.25	1.8	72.12	1.9	66.77	2.2
0.15	3	0.7	6.88	2.49	68.06	2.3	71.25	1.8	72.12	1.9	66.77	2.2
0.25	3	0.7	5.77	2.3	69.38	2.2	72.65	2.1	75.64	2.1	71.06	2.2
0.40	3	0.71	5.90	2.43	75.55	2.0	69.63	2.0	67.1	2.0	76.3	2.0

* randomly generated

TABLE 5B.6: Results of ELECTRE I(α)(σ) + ELECTRE I(α)(σ) Simulation Runs

α	σ	C	Prob.	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
0.4	10	2	0.51	7.85	2.27	65.27	2.0	63.99	2.1	77.77	1.8	67.98	2.1
0.4	10	3	0.67	6.39	2.2	67.43	2.2	80	1.6	68.12	2.0	69.08	2.3

* randomly generated

APPENDIX 5C: THE DETAILED RESULTS OF THE EFFECTS THAT EXTERNAL FACTORS HAVE ON THE VARIOUS MCDM APPROACHES FOR THE SECOND DATA SET

This appendix provides the tabulated results of the effects that the three external factors have on the solution produced by the four MCDM approaches. The shortened column headings of these tables are explained in appendix 4A at the end of chapter 4, with the exception of the following:

Pol. El's.: The number of policy elements (m).

Der. Atts.: The number of derived attributes (n).

Int.: The number of interest groups (p).

The tables of results for the investigation of the effects of these external factors are provided on the next 6 pages.

TABLE 5C.1: Results of Maxscoresum + Maxscoresum Simulation Runs

Pol. El's.	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
4	3	0.61	3.2	1	75.33	2.5	74.22	2.3	78.14	2.7	76.68	2.5
5	3	0.62	3.58	1	72.28	2.1	71.36	2.1	76.89	2.2	72.26	2.0
6	3	0.5	5.68	1.01	79.2	2.1	75.68	2.4	74.5	2.5	75.36	2.7
4	4	0.65	2.61	1	80.49	2.4	76.63	2.3	78.94	2.2	75.52	2.4
5	4	0.6	4.75	1	74.18	2.3	75.77	2.2	73.19	2.3	74.52	2.1
6	4	0.68	4.08	1	79.56	2.2	77.65	2.4	80.25	2.0	77.89	2.2

* randomly generated

TABLE 5C.2: Results of Minranksum + Minranksum Simulation Runs

Pol. El's.	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
4	3	0.63	4.08	1.19	75.3	2.6	74.85	2.2	70.77	2.4	78.06	2.0
5	3	0.56	5.03	1.25	74.34	2.3	73.79	2.2	71.06	1.9	70.92	2.3
6	3	0.49	6.16	1.3	77.84	2.0	76.44	2.7	75.27	2.5	73.85	2.5
4	4	0.69	1.29	3.84	74.54	2.6	78.61	2.2	71.77	2.4	74.48	2.4
5	4	0.52	4.75	1.29	70.22	2.3	73.65	2.3	69.26	2.1	76.76	1.9
6	4	0.53	6.84	1.23	80.24	2.0	76.4	2.4	73.87	2.2	72.47	2.7

* randomly generated

TABLE 5C.3: Results of Max/Min + Max/Min Simulation Runs

Pol. El's.	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
4	3	0.53	2.37	1	73.21	1.8	73.68	1.8	71.45	1.9	71.46	2.0
5	3	0.54	2.43	1	71.11	1.7	71.57	1.7	74.51	1.6	71.42	1.4
6	3	0.49	2.97	1.07	75.38	1.6	72.78	1.9	76.51	1.8	74.26	1.8
4	4	0.51	2.41	1	72.82	1.9	70.99	1.8	73.69	1.8	72.55	1.8
5	4	0.49	2.65	1	70.11	1.6	71.26	1.5	70.77	1.7	71.92	1.6
6	4	0.41	3.69	1	73.14	1.9	72.97	1.9	72.61	1.8	76.77	1.6

* randomly generated

TABLE 5C.4: Results of ELECTRE I + ELECTRE I Simulation Runs

Pol. El's.	C	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
4	2	0.59	5.53	1.72	74.21	2.4	67.91	2.7	76.15	2.0	71.92	2.3
5	2	0.61	6.85	1.87	73.44	2.1	70.25	2.3	71.86	2.0	70.58	2.2
6	2	0.56	9.63	1.84	74.27	2.1	71.69	2.4	75.39	2.1	71.11	2.2
4	3	0.69	4.93	1.78	78.68	2.0	73.28	2.1	71.08	2.3	74.44	2.3
5	3	0.57	6.93	2.01	71.02	2.0	72.17	2.2	69.89	2.3	69.79	2.1
6	3	0.6	9.58	1.92	75.11	2.1	73.05	2.2	78.52	1.7	70.96	2.4

* randomly generated

TABLE 5C.5 Results of Maxscoresum + Maxscoresum Simulation Runs

Der. Atts.	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
4	3	0.61	3.2	1	75.33	2.5	74.22	2.3	78.14	2.7	76.68	2.5
5	3	0.64	2.37	1	79.44	2.2	76.55	2.3	72.77	2.4	72.91	2.3
6	3	0.61	3.54	1	79.2	2.2	76.17	2.5	72.96	2.6	74.98	2.6
4	4	0.65	2.61	1	80.49	2.4	76.63	2.3	78.94	2.2	75.52	2.4
5	4	0.68	2.85	1	77.99	1.9	75.42	2.6	74.59	2.3	74.17	2.5
6	4	0.67	2.74	1	72.85	2.6	78.14	2.2	77.51	2.3	78.79	2.0

* randomly generated

TABLE 5C.6: Results of Minranksum + Minranksum Simulation Runs

Der. Atts.	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
4	3	0.63	4.08	1.19	75.3	2.6	74.85	2.2	70.77	2.4	78.06	2.0
5	3	0.53	3.47	1.19	72.31	2.3	73.59	2.5	76.41	2.3	73.86	2.4
6	3	0.68	3.76	1.3	76.72	2.1	74.73	2.5	74.64	2.3	77.06	2.5
4	4	0.69	1.29	3.84	74.54	2.6	78.61	2.2	71.77	2.4	74.48	2.4
5	4	0.56	4.31	1.19	77.93	2.3	71.12	2.3	75.48	2.4	69.53	2.4
6	4	0.64	3.3	1.3	75.3	2.3	78.64	2.1	76.24	2.5	76.33	2.3

* randomly generated

TABLE 5C.7: Results of Max/Min + Max/Min Simulation Runs

Der. Atts.	K	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
4	3	0.53	2.37	1	73.21	1.8	73.68	1.8	71.45	1.9	71.46	2.0
5	3	0.4	2.87	1	69.41	1.8	72.5	1.8	69.79	1.8	69.82	1.9
6	3	0.49	2.44	1	72.15	2.0	74.51	1.8	73.46	2.0	73.55	1.8
4	4	0.51	2.41	1	72.82	1.9	70.99	1.8	73.69	1.8	72.55	1.8
5	4	0.46	2.65	1	70.71	1.8	72.14	1.8	70.44	1.7	72.01	1.8
6	4	0.56	2.03	1	73.17	1.9	74.45	2.0	72.94	1.9	74.02	1.7

* randomly generated

TABLE 5C.8: Results of ELECTRE I + ELECTRE I Simulation Runs

Der. Atts.	C	Probability	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4) Interest Groups* and their associated Standard Deviations: Combination of Methods							
4	2	0.59	5.53	1.72	74.21	2.4	67.91	2.7	76.15	2.0	71.92	2.3
5	2	0.67	5.65	1.85	71.72	2.2	71.25	2.1	73.56	2.2	70.65	2.2
6	2	0.63	6.03	1.75	68.73	2.5	73.71	2.3	75.06	2.4	75.23	2.3
4	3	0.69	4.93	1.78	78.68	2.0	73.28	2.1	71.08	2.3	74.44	2.3
5	3	0.68	6.08	1.88	71.19	2.1	68.97	2.5	70.41	2.4	76.32	2.1
6	3	0.74	5.02	1.68	71.96	2.2	72.25	2.2	76.72	1.8	70.54	2.3

* randomly generated

TABLE 5C.9: Results of Maxscoresum + Maxscoresum Simulation Runs

Int.	K	Prob.	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4/5/6) Interest Groups* and their associated Standard Deviations: Combination of Methods											
4	3	0.61	3.2	1	75.33	2.5	74.22	2.3	78.14	2.7	76.68	2.5				
5	3	0.6	5	1	70.4	2.9	73.37	2.7	73.24	2.6	73.48	2.4	75.88	2.3		
6	3	0.51	5.76	1	70.14	3.0	72.81	2.5	74.19	2.8	69.6	2.8	72.16	2.7	74.94	2.6
4	4	0.65	2.61	1	80.49	2.4	76.63	2.3	78.94	2.2	75.52	2.4				
5	4	0.56	4.17	1	73.91	2.7	68.74	2.9	71.41	2.8	71.78	2.8	78.19	2.4		
6	4	0.57	4.82	1	70.8	2.9	71.83	2.6	73.04	2.8	73.25	2.3	74.02	2.5	70.29	2.9

* randomly generated

TABLE 5C.10: Results of Minranksum + Minranksum Simulation Runs

Int.	K	Prob.	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4/5/6) Interest Groups* and their associated Standard Deviations: Combination of Methods											
4	3	0.63	4.08	1.19	75.3	2.6	74.85	2.2	70.77	2.4	78.06	2.0				
5	3	0.51	5.46	1.18	72.91	2.7	74.75	2.5	68.55	2.7	68.51	2.9	74.4	2.4		
6	3	0.56	5.52	1.3	68.51	2.4	68.96	2.6	74.83	2.4	68.44	2.6	69.36	2.6	72.69	2.2
4	4	0.69	1.29	3.84	74.54	2.6	78.61	2.2	71.77	2.4	74.48	2.4				
5	4	0.48	5.98	1.26	71.21	2.9	77.44	2.4	71.18	2.6	69.3	2.5	71.51	2.6		
6	4	0.41	6.79	1.21	68.91	2.8	68.2	2.8	69.61	2.9	72.69	2.6	70.94	2.9	70.01	2.9

* randomly generated

TABLE 5C.11: Results of Max/Min + Max/Min Simulation Runs

Int.	K	Prob.	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4/5/6) Interest Groups* and their associated Standard Deviations: Combination of Methods											
4	3	0.53	2.37	1	73.21	1.8	73.68	1.8	71.45	1.9	71.46	2.0				
5	3	0.43	2.55	1	68.86	1.7	70.15	1.9	69.67	1.7	70.18	1.8	68.26	1.7		
6	3	0.42	2.75	1	65.76	2.0	66.07	1.9	67.46	1.7	69.45	2.0	68.37	2.0	63.51	1.9
4	4	0.51	2.41	1	72.82	1.9	70.99	1.8	73.69	1.8	72.55	1.8				
5	4	0.33	2.75	1	68.21	1.9	67.18	1.9	69.53	1.6	67.32	2.0	66.7	1.8		
6	4	0.31	2.96	1	66.35	1.5	66.58	1.9	66.38	1.8	64.22	1.8	67.31	1.6	68.5	1.6

* randomly generated

TABLE 5C.12: Results of ELECTRE I + ELECTRE I Simulation Runs

Int.	C	Prob.	Average Rank	Soln. Set Size	2. Mean Utility Values for the (4/5/6) Interest Groups* and their associated Standard Deviations: Combination of Methods											
4	2	0.59	5.53	1.72	74.21	2.4	67.91	2.7	76.15	2.0	71.92	2.3				
5	2	0.72	6.24	2.31	73.47	2.0	72.14	2.2	67.34	2.2	72.51	2.0	72.01	2.1		
6	2	0.61	8.48	2.57	66.21	2.2	67.18	2.1	60.92	2.3	71.79	2.1	66.47	2.1	68.73	2.3
4	3	0.69	4.93	1.78	78.68	2.0	73.28	2.1	71.08	2.3	74.44	2.3				
5	3	0.86	6.43	2.78	69.69	1.9	72.63	1.6	67.95	1.8	68.02	2.0	72.32	1.8		
6	3	0.73	8.02	2.57	65.53	2.0	67.8	2.2	70.26	2.2	67.15	2.2	67.58	2.2	68.97	2.1

* randomly generated

CHAPTER 6: CONCLUSIONS

We have previously stated that the literature does not deal explicitly with the two areas of focus, forming part of figure 2.1, on which this paper concentrated. The MCDM and related methodologies were therefore analyzed with a view of finding suitable methods, perhaps requiring a degree of adaptation, that could be implemented in order to:

(1) filter a large set of alternatives (analogous to our Background Set) to form a smaller and more manageable Foreground Set (on which direct value judgements could be made), and

(2) further refine and ultimately reduce the Foreground Set to a solution set of alternatives.

A main criterion that was required of the MCDM and related methods was that they be both interactive and iterative. The first criterion would ensure that the DMs were not only aware of the workings of the method (i.e. avoiding a "black box" type of approach), but that they would participate in and provide input on a continual basis. Furthermore, the ability to revise a particular outcome produced by the method was also a criterion that was placed on the particular MCDM approach, and this would therefore require the MCDM method to be iterative in nature.

The MCDM and related approaches were also rated on the quality of the solutions they produced. This was achieved by observing the proximity of the solution produced by the MCDM method to that of the Nash optimum solution, which we used as a benchmark in our study. We were also interested to see how consistent and reliable the solutions were that were produced by the various combinations of MCDM methods.

As far as the analysis of the methods is concerned, we reported the results obtained from the simulation studies in chapters 4 and 5. Our overall conclusions, based on the outcomes of these simulation studies as well as the criteria we have mentioned in the previous paragraph, are provided in the two sections of this chapter below.

6.1 Specific to the Simulation Study

Based on the outcomes of the MCDM approaches we have implemented in the simulation studies, as reported in chapters 4 and 5, the following conclusions may be drawn:

(1) For the analysis conducted on the first data set in chapter 4, we tended to favour the Max/Min + Max/Min and the Maxscoresum + Maxscoresum MCDM approaches. This was considered from an overall point of view, wherein we looked at both the quality and robustness of the solution set produced. We did feel that the reason why the Maxscoresum + Maxscoresum approach failed to be robust for large forms of precision error being made on the thermometer scores needed further investigation.

(2) For the analysis conducted on the second data set in chapter 5, we again tended to favour the Max/Min + Max/Min and Maxscoresum + Maxscoresum approaches. We again considered both the quality and robustness of the solution produced. For this data set, we recommended that both the Max/Min + Max/Min and Maxscoresum + Maxscoresum approaches be further investigated, since the latter failed to be robust to precision errors being made on the thermometer scores, whilst the first mentioned has a low probability of selecting the Nash optimum as the compromise solution.

(3) The analysis on the second data set also looked at the effects that external factors (*viz.* the number of policy elements, the number of derived attributes and the number of interest groups) have on the solution set produced. We found that for the Max/Min + Max/Min MCDM approach, the number of policy elements did not greatly affect the results produced by this combination of methods. For the Maxscoresum + Maxscoresum approach, the number of derived attributes did not affect the results produced by this combination of MCDM methods.

(4) Overall, we could therefore conclude that the Max/Min + Max/Min and the Maxscoresum + Maxscoresum MCDM approaches seem to be the two methods that provided us with the most reliable solutions. Both these combinations of methods tend to produce single best policy scenarios as their solutions. They also produce a complete rank ordering of the remaining alternatives in the (revised) Foreground Set. This allows the DMs the freedom of choice as to decide whether they will settle for a single best alternative or include a (subjective) number of the next best ranked alternatives to consider for further investigation. This flexibility therefore adds to the attractive nature of these two combinations of methods.

6.2 Overall Value to Multicriteria Policy Planning

Based on the literature we have surveyed, the following overall conclusions may be drawn:

(1) It would most certainly be advantageous to apply screening policies in order to reduce the number of alternatives under consideration, before attempting to reduce the Background Set to a more manageable smaller set.

(2) The utility based approach in which alternatives are directly compared in terms of each criterion, was found to be of particular value when a small number of alternatives, such as in the Foreground Set, were being considered. Implementation, however, becomes tedious when a large number of alternatives is being considered. The approach does provide considerable insight into the planning procedure for the various DMs and this is particularly the case when value functions are used in an interactive sense.

Value functions in interactive mode allow DMs to be involved in the process on a continual basis. The approach of Korhonen, Wallenius and Zionts (1984), which we used in the simulation study to revise the Foreground Set, was found to be well suited for this purpose. When compared to conventional MAUT, value functions in interactive mode also allow for a reduced amount of comparisons to be made between various alternatives. In this regard they therefore lend themselves well to the task of reducing the large Background Set to a smaller Foreground Set of alternatives.

(3) Interactive (multiple) goal programming approaches are well suited to the process of reducing the Background Set of alternatives. The Wierzbicki (1980) scalarizing function was used in the simulation study as part of the Steuer/Wierzbicki method to perform this task. We found that it worked well and provided us with a reasonable spread (i.e. including the extreme and in-between attribute values on which these alternatives are formulated) of scenarios to form our Foreground Set.

(4) The VISA approach of Belton and Vickers (1989) seems to be well suited to aiding group decision support. The presentation of information (i.e. how conflicting interest groups may rate or score various alternatives) becomes important and the "thermometer" type scales (also available in VISA) certainly provides a good visual representation for scoring the various alternatives. It is particularly effective when the number of alternatives is relatively

small, for instance when analysing the Foreground Set. Stewart *et al.* (1993) report that "people from a wide variety of backgrounds" were not only capable but also confident when they compared alternatives in a Foreground Set environment on the basis of various criteria by using "thermometer scales".

(5) Overall, the appropriate MCDM and related approaches we have reviewed and implemented in the simulation studies, still need further refining and some degree of adaptation for use in an interactive DSS. This is perhaps the case since most of these methods have in this study been implemented in a (hypothetical) environment, that not only requires direct value judgements to be made on various societal interests, but also requires judgements to be made of a less tangible nature. These judgements will impact on the benefits to society as a whole and it is not always possible to express or formulate them in a quantifiable manner.

However, the MCDM methods we have reviewed all require that these judgements be expressed in a quantifiable manner, so that they can be used when the alternative policy outcomes are being evaluated. We therefore have to accommodate this difference where the expression of these judgements is concerned and incorporate this into the overall scenario based planning procedure.

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